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Estimating Central Bank Reaction Functions with Ordered Probit:

A Note

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Abstract

In a seminal paper, Eichengreen et al. (1985) proposed a strategy to estimate monetary reaction functions using dynamic probit techniques to allow for the discreteness of interest rate changes. We follow the approach in the literature on dynamic Tayor rules and derive a monetary policy reaction function that can be estimated by standard ordered probit methods. We contrast the two specifications and illustrate our technique on the data set used by Eichengreen et al. to study interest rate setting by the Bank of England in 1925-1931.

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I. Introduction.

In 1993 John Taylor suggested that US monetary policy in the late 1980s and early 1990s was well captured by a rule of thumb according to which the federal funds target was changed more than one-for-one to changes in inflation and by some fraction of changes in the output gap in the economy. A literature has subsequently emerged which evaluates the performance of central banks with reference to Taylor's rule. This involves estimating a reaction function for the interest rate under the control of the policy authorities. Focusing on the central bank's policy rate raises an econometric problem because this rate is altered relatively infrequently, and when it is altered, the change is by a discrete amount, usually a multiple of twenty-five basis points. The seminal paper by Eichengreen, Watson, and Grossman (1985), EWG hereafter, proposed a probit model to deal with this problem. Furthermore, because of the serial correlation in the policy rate, they argued that it is necessary to use a dynamic version of the probit model.[1] This model is technically difficult to implement. In this note we argue that recent work on dynamic Taylor rules suggest a method to deal with serial correlation that permits the use of the "standard" ordered probit model.[2] We use the data from EWG to implement our suggestion.

II. Modeling discrete changes in policy interest rates.

The approach of EWG can be illustrated as follows. Consider the following definitions:

i_t = the policy interest rates determined by the central bank

i_t^* = the optimal level of the policy interest rate if it could be set on a continuous scale

[1] Davutyan and Park (1995) employ the EWG model in their study of the operations of the Bank of England prior to World War I.

[2] Recent studies estimating central bank reaction functions using ordered probit (or logit) models include Christensen (2003), Dueker (1999), Galí et al. (2003), Gerlach (2004), Heinemann and Hüfner (2002) and Kim et al. (2004).

$d =$ the minimum discrete amount that the policy rate can be adjusted, i.e. the only feasible adjustments in the policy rate are $\Delta i_t = \{0, \pm d, \pm 2d, \pm 3d, \text{etc.}\}$

Since the minimum change in the policy rate is d, the central bank will only change it if the optimal level of the interest rate is far enough from its level last period. Let p_i (n_i) be the minimum distance that justifies positive (negative) change in the interest rate of size i·d {i=1, 2, …}. Then, for i=1,2

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\begin{aligned}
\Delta i_t &= 2d && \text{if} && i_{t-1} + p_2 < i_t^* < i_{t-1} + p_3 \\
\Delta i_t &= d && \text{if} && i_{t-1} + p_1 < i_t^* < i_{t-1} + p_2 \\
\Delta i_t &= 0 && \text{if} && i_{t-1} - n_1 < i_t^* < i_{t-1} + p_1 \qquad (1) \\
\Delta i_t &= -d && \text{if} && i_{t-1} - n_2 < i_t^* < i_{t-1} - n_1 \\
\Delta i_t &= -2d && \text{if} && i_{t-1} - n_3 < i_t^* < i_{t-1} - n_2
\end{aligned}
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This specification lends itself to empirical analysis using ordered probit methods once the determinants of i_t^* have been identified. EWR proposed a model in which the change in i_t^* was a function of a vector of variables, z_t, describing the state of the economy in time t as in (2).

$$\Delta i_t^* = z_t \beta + u_t \qquad (2)$$

Since this equation determines the change in i_t^* whereas its level belongs in (1), the usual ordered probit model is not appropriate. The reason is that the boundaries at which changes in the policy rate are triggered depend on the unobserved i_{t-1}^*. This requires the use of a dynamic ordered probit model which is computationally onerous and not readily available in standard estimation software.[3]

[3] The estimation difficulties can be illustrated by eliminating past values of the optimal interest rates in (2) through iterative substitution. This yields $i_t^* = \left(\sum_{j=1}^{t} z_j \right) \beta + \sum_{j=1}^{t} u_j + i_0^*$.

An alternative model of the determinants of i_t^* can be derived from the recent literature on dynamic Taylor rules. We start from a linear static reaction function of the form

$$i_t^{TR} = x_t\beta + v_t \quad (3)$$

where i_t^{TR} represents the interest rate implied by a static Taylor rule. Following Judd and Rudebusch (1998), we assume gradual adjustment of optimal the policy rate of the form:

$$i_t^* - i_{t-1} = \lambda_0\left(i_t^{TR} - i_{t-1}\right) + \lambda_1 \Delta i_{t-1} \quad (4)$$

Using (3) and (4) we obtain the expression for the optimal interest rate:

$$i_t^* = (1-\lambda_0)i_{t-1} + \lambda_0 x_t\beta + \lambda_1 \Delta i_{t-1} + \lambda_0 v_t \quad (5)$$

The specification in (5) has the advantage that since all regressors are observed, it permits the use of the "standard" ordered probit model in estimation which is computationally relatively simple. By including the lagged value of the policy rate among the explanatory variables this specification is also potentially able to deal with the problem of serial correlation which motivated the estimation methods chosen in EWR. On a priori grounds it is not possible to determine which of (2) or (5) is superior. Occam's razor argues in favor of (5), but it must be able to overcome the serial correlation problem mentioned by EWR. Whether it can is an empirical question, and the answer may be different from case to case. In the following section we implement our alternative specification using the original data in EWR and show that in this case at least it gives plausible results and pass the test for serially correlation.

III. Bank Rate Policy Under the Interwar Gold Standard: An Illustration.

We follow EWG who studied the setting of Bank Rate by the Bank of England during the Gold Standard period 1925-1931.[4] To briefly summarise that study, it found that the

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The dependence of i_t^* on the entire history of z and u is what complicates the construction of the likelihood function.

[4] This data is available on Mark Watson's home page at www.wws.princeton.edu/~mwatson.

Bank of England's decision to change interest rates could be tied to a limited number of variables:

- Weekly losses of gold reserves (by contrast, weekly gains of gold reserves were not significant in their model).
- The spread between Bank Rate and the rate on London 90-day bankers drafts. EWG found that the Bank of England was particularly concerned by this interest rate spread if it was increasing.
- The level of Bank Rate. In particular, the Bank attempted to reduce Bank Rate if it exceeded 4%.

By contrast, EWG found only weak evidence that the Bank of England responded to movements in unemployment, the volume of Treasury bill issuance, the Sterling – US dollar exchange rate, and the spread between London and New York interest rates, as captured by 90-day bankers acceptances in New York.

We start our specification from one that is loosely modeled on that of EWG, except that we incorporate the change in New York interest rates, as well as the lagged level and change of Bank Rate.[5] After deleting insignificant variables, we obtained the following model:

$$\Pr ob(\Delta BR) = \alpha_0 \Delta i_t^{NY} + \alpha_1 \Delta G_t^- + \alpha_2 \Delta^8 G_{t-1} + \alpha_3 \left(BR_{t-1} - i_{t-1}^L\right) + \dots \\ + \alpha_4 \Delta BR_{t-1} + \alpha_5 BR_{t-1} + v_t$$

where BR_t denotes Bank Rate, i_t^L (i_t^{NY}) market rates in London (New York), ΔG_t^- the weekly decline in the Bank of England's gold reserves, and $\Delta^8 G_t$ the change in the Bank's gold reserves over 8 weeks.

The estimates in Table 1 indicate that all parameters are highly significant and have the expected signs. We also report the value of the likelihood function, McFadden's

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[5] Also, in order to ensure that the regressors are not correlated by construction with decisions to change interest rates, we do not use any regressors based on the current value of Bank Rate.

likelihood ratio index (or "$\text{pseudo} - R^2$"), and the p-value from a test of the hypothesis of no first-order serial correlation in the residuals, constructed as suggested by Gourieroux et al. (1985, p. 326).

Overall the results are highly supportive of the analysis in EWG and we do not comment on them further. The main novelty is that the change in New York rates in appears to have influenced the Bank of England's interest rate decisions, perhaps because this variable captures international financial factors. The importance of market conditions in New York is readily apparent from Figure 1 that shows how New York rates rose in 1927-1929, and fell from 1929 onwards, in advance of London rates. We also finding that the lagged level and change of Bank Rate are highly significant, as suggested by the model of interest rate setting that we proposed above.

IV. Conclusions.

Using recent work on dynamic Taylor rules we propose a specification of a reaction function of a central bank that permits the use of the ordinary ordered probit model for to estimate discrete-valued policy rate changes instead of the more difficult dynamic probit model. We illustrate the approach by applying it to the data used in the seminal paper of Eichengreen, Watson, and Grossman to study the Bank Rate policy of the Bank of England under the interwar gold standard. Overall our results are consistent with those reported in EWG, and a diagnostic test for serial correlation suggests that our simpler estimation procedure is justified.

References

Carstensen, Kai (2003), "Estimating the ECB policy reaction function," forthcoming in Empirical Economics.

Davutyan, Nurham and William R. Park (1995), "The operations of the Bank of England, 1880-1908: A dynamic probit approach," Journal of Money, Credit, and Banking, 27, 1099-1112.

Dueker, Michael (1999), "Measuring monetary policy inertia in target fed funds rate changes," Federal Reserve Bank of St Louis Review, September/October, 3-9.

Eichengreen, Barry, Mark Watson and Richard S. Grossman (1985), "Bank rate policy under the interwar gold standard: A dynamic probit model," Economic Journal, 95, 725-745.

Galí, Jordi, Stefan Gerlach, Julio Rotemberg, Harald Uhlig and Michael Woodford (2003), *The monetary policy strategy of the ECB reconsidered: Monitoring the European Central Bank 5*. CEPR.

Gerlach, Stefan (2004), "Interest rate setting by the ECB: Words and deeds," unpublished paper.

Gourieroux, Christian, Alain Monfort and Alain Trognon (1985), "A general approach to serial correlation," Econometric Theory, 1, 315-340.

Heinemann, Friedrich and Felix Hüfner (2002), "Is the view from the Eurotower purely European? National divergence and ECB interest rate policy," Scottish Journal of Political Economy, 51, 544-558.

Judd, John P. and Glenn D. Rudebusch (1998), "Taylor's rule and the fed: 1970-1997," Federal Reserve Bank of San Francisco Economic Review, No. 3, 3-16.

Kim, Tae-Hwan Paul Mizen and Alan Thanaset (2004), "Predicting Directional Changes in the UK Interest Rate: The Usefulness of Information from the Taylor Rule Versus a Wider Alternative," Unpublished paper.

Taylor, John (1993), "Discretion versus policy rules in practice," Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

Table 1

Estimates for June 1926 – September 1931

328 observations

Change in NY rate	4.970 [0.000]
Decline in Gold Reserves	-0.255 [0.000]
Change in Gold Reserves over 8 weeks, lagged	-0.047 [0.007]
Spread between Bank Rate and London market rate, lagged	-2.469 [0.001]
Change in Bank Rate, lagged	-2.248 [0.019]
Bank rate, lagged	-0.345 [0.064]
Log likelihood	-47.506
Pseudo R-squared	0.411
Serial correlation	[0.438]

Note: p-values in parenthesis.

Figure 1
Bank Rate and Market Rates in London and New York