

OUTPUT GAPS AND INFLATION:
Unobservable-components estimates for the G-7 countries

by
Stefan Gerlach and Frank Smets*

Bank for International Settlements
CH-4002 Basle
Switzerland

Abstract

This paper uses unobservable components techniques to estimate output gaps in the G-7 countries. Actual output is the sum of potential output, which follows a random walk with a potentially stochastically varying rate of drift, and the output gap, which obeys an AR(2)-process. The output gap also affects the change in inflation, so that output and inflation both contain information about the output gap. Our preliminary estimates of the model suggests that UC-techniques may usefully be applied to the estimation of output gaps. The paper ends with a brief discussion of a number of possible extensions of the model.

* The views expressed in this paper are solely our own and not necessarily those of the BIS. We are grateful to Florence Béranger for help with assembling the data set. Address: BIS, CH-4002 Basle, Switzerland, tel -41-61-280.8080, fax: -41-61-280.9100.

1. Introduction

The output gap -- the discrepancy between actual and potential output -- plays a central role in determining the behaviour of wages and prices. There is a considerable body of evidence indicating that inflationary pressures tend to rise when output is above, and fall when actual output is below, potential.¹ In light of its role in the inflation process, the output gap is of critical importance for the conduct of monetary policy, particularly for those central banks that gear policy directly to the ultimate target of price stability without the use of an intermediate target. Thus, by changing monetary conditions when output threatens to deviate from potential, central banks can balance aggregate demand for goods and services against productive capacity and in this fashion influence the inflation process.

One problem in this context is that data on potential output are not directly available, so that policy must be conducted on the basis of estimates of the output gap. For this reason, much research has been devoted to the empirical modelling of potential output. Several different research strategies have been followed. At the risk of some oversimplification, it is useful to distinguish between three approaches: (i) *statistical* approaches, (ii) *structural* approaches, and (iii) *mixed* approaches.

The first approach views the estimation problem solely as a statistical exercise in which potential output is estimated from data on actual output. One particularly simple way to do this is to regress the log of output on a constant and time trend, and to interpret the fitted value as potential output or, equivalently, the residuals as the output gap. The widely used Hodrick-Prescott (HP) filter (1980) and the time-series approaches employed by Beveridge and Nelson (1981), Watson (1986) and Clark (1989) are other examples of work of this genre. One drawback with the statistical approach is that it does not incorporate other information, such as data on the state of the labour market or the rate of inflation, into the analysis despite the fact that such data are likely to contain information about potential output or, equivalently, the output gap. Of course, to the extent that such structural relationships are difficult to formalise exactly and therefore a potential source of specification error, the fact that statistical approaches obviate the need for such relationships may also be attractive.

The structural approach, in contrast, exploits economic theory to estimate potential output. The common methodology is to use a production function (most often of the Cobb-Douglas form) to relate potential output to various input factors such as labour, capital and productivity.² While the use of economic theory to guide the estimation process is attractive, the data requirements (in particular with respect to the construction of capital

¹ Following the original evidence of Phillips (1958), a large number of papers have documented the relationship between measures of capacity utilisation and subsequent inflation. For an early contribution, see Gordon (1970). Recent evidence for the United States is surveyed in Fuhrer (1995).

² See OECD Economic Outlook (1994). Another example of the structural approach involves using Okun's law (e.g. Braun 1990).

stocks) can be considerable. Furthermore, labour market and productivity data, which are necessary for the production function approach, are also subject to cyclical influences, which need to be removed to construct estimates of potential output. As a result, the important question concerning trend versus cyclical behaviour is merely shifted to these variables.³ Another drawback of this approach is that it is difficult to construct confidence bands for estimated output gaps.

The mixed approach combines elements of both the statistical and structural approach by using time series methods to study output data together with data from other economic variables, which theory suggests are related to potential output or the output gap. Combining and jointly estimating a statistical model of actual and potential output with more structural equations such as the Phillips curve is attractive for at least two reasons. First, from a conceptual perspective it is desirable to add structural equations to give the notion of an output gap an economic, as opposed to a statistical content. As the usefulness of the output gap for the conduct of monetary policy lies in the first place in its causal relationship with future inflation and the implications thereof for the direction of monetary policy changes, it seems in particular appropriate to combine the statistical model of the output gap with a Phillips-curve relationship. Second, the short-run Phillips curve implies that changes in the rate of inflation will contain information about the output gap. Thus, using both inflation and output data in estimation permits more precise estimates of the output gap to be made.

One example of the mixed approach which uses data on both output and inflation is the extension of the univariate HP filter proposed by Laxton and Tetlow (1992), which allows other information on, for instance, inflation and unemployment to be used to sharpen the identification of potential output.⁴ A second example is the unobservable-components (UC) approach employed by Kuttner (1994) to estimate potential GDP in the United States, which combines Watson's (1986) univariate statistical model with a Phillips-curve type relationship which links the output gap to changes in inflation.

In this paper we use the UC approach to estimate the output gap, the growth rate of potential output, and the slope of the short-run Phillips curve in Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States, using quarterly data starting (depending on the country) from the early 1960s or 1970s. Our choice of the UC approach rather than the multivariate HP filter as research methodology stems from the fact that a number of authors have noted that the HP filter is associated with several shortcomings.⁵

First, as noted by King and Rebelo (1993) and Harvey and Jaeger (1993), the HP filter can be thought of as the solution to a signal extraction problem in which the researcher

³ For instance, there are both statistical and structural approaches to estimating the natural rate of unemployment.

⁴ For a recent application of this approach to the G-7 countries, see Haltmaier (1996).

⁵ See King and Rebelo (1993), Harvey and Jaeger (1993) and St-Amant and van Norden (1996).

attempts to decompose a time series into its trend and cyclical component. However, the HP filter's ability to do so accurately, depends on the specifics of the underlying stochastic model and, hence, will vary over different data sets.⁶ It seems thus more appropriate to estimate the underlying statistical model directly using UC techniques.

Second, applying the HP filter involves choosing a weight which determines the smoothness of the trend component. In the extension of the univariate HP filter by Laxton and Tetlow (1992) this arbitrariness also applies to the weight put on the additional information. In contrast, the UC approach lets the data decide on the relative importance of the various shocks to the system.

Third, as forcefully demonstrated by St-Amant and van Norden (1996), the HP filter is subject to the problem that, at the end of the sample, the estimate of potential output will be biased towards the most recent observations. This end-point problem is particularly worrisome from a policy perspective because for the formulation of monetary policy the most recent estimates of the output gap are the most relevant.

The rest of the paper is structured as follows. In Section 2 we briefly outline our empirical model, which is closely related to the models estimated by Watson (1986), Clark (1989), and Kuttner (1994). In this model, actual output is assumed to be the sum of potential output and the output gap. In contrast to Watson (1986) and Kuttner (1994) who assume that potential output follows a random walk with a constant drift, which seems appropriate in the case of the United States, we follow Clark (1989) by also allowing the drift to follow a random walk. This is necessary to capture the more pronounced shifts in the rate of potential growth that occurred in some of the other G-7 countries. The output gap is assumed to follow an AR(2)-process. Finally, we follow Kuttner (1994) in assuming that the output gap affects the change in inflation. This assumption, which is based on an expectations-augmented Phillips curve, implies that output and inflation both contain information about the unobserved output gap. We end this section by discussing how the model can be put into state-space form, and how maximum likelihood estimates can be obtained using Kalman filtering.

In Section 3 we turn to the estimates of the basic Watson-Clark-Kuttner (WCK) model. We estimate it using quarterly data on real GDP and CPI inflation from the G-7 countries. The results suggest that the WCK model can be usefully applied to the other G-7 countries. With the exception of Germany, we find that in all countries the stochastic proces

⁶ King and Rebelo (1993) and Harvey and Jaeger (1993) demonstrate that the HP filter is optimal when potential output obeys a random walk in which the drift term also follows a random walk, and the output gap, paradoxically, is white noise. They also show that while the filter works quite well for real GDP in the United States, it will fail to accurately extract the cyclical component of many other time series. St-Amant and van Norden (1996 p.9) summarise the conditions required for the HP filter to adequately identify the cyclical component as: "the spectrum of the original series must have a peak located at business-cycle frequencies, which must account for an important part of the variance of the series".

for the output gap is well captured by an AR(2) model with the first AR parameter significantly larger than one and the second parameter significantly negative. This implies that, as previously found for the United States, the typical response of the output gap to a shock is hump-shaped. In all countries, the slope of the short-run Phillips curve is positive, and, with the exception of France, significantly different from zero. Typically, a 1 percentage point increase in the output gap raises annualised inflation by about 15 to 30 basis points.⁷

In Section 4 we discuss possible extensions, which deal primarily with the specification of the inflation equation. Focusing on the model for Canada, we test for the significance of additional exogenous variables such as terms-of-trade and oil price shocks in the inflation equation. We also allow for the presence of acceleration effects of the output gap on inflation. In addition, we check how robust our previous estimate of the output gap is to these changes. In the next draft of this paper, we hope to do this for the other G7 countries. Finally, Section 5 contains some conclusions.

2. The Watson-Clark-Kuttner model

In this section we present the WCK-model. In the first two sections below, we present the output and inflation equations in turn. Section 2.3 shows how the model can be written in state-space form and estimated using a Kalman-filter.

2.1. Output

The specification of the process for output follows Watson (1986). Let y_t , y_t^p and z_t denote the logarithms of actual and potential real output, and the output gap, respectively. Of these variables, only actual output is observed: the purpose of the exercise that follows below is to estimate the latter two. Formally, we have that:

$$(1) \quad y_t \equiv y_t^p + z_t.$$

Furthermore, potential output is assumed to follow a random walk with drift:

$$(2) \quad y_t^p = \mu_t + y_{t-1}^p + \varepsilon_t^y,$$

where $\varepsilon_t^y \sim N(0, \sigma_y^2)$. Watson (1986) and Kuttner (1994) both assume that the drift term is constant over time ($\mu_t = \mu_{t-1}$). While this assumption seems suitable for data for the United States, Clark (1989) shows that it imposes considerable violence on the data for several other countries, notably France and Japan, where pronounced changes in the trend growth rate of

⁷ Exceptions are Japan and the United Kingdom, where the estimated inflation effects are substantially larger.

output have taken place. We therefore follow Clark (1989) by assuming that the drift term also follows a random walk:

$$(3) \quad \mu_t = \mu_{t-1} + \varepsilon_t^\mu,$$

where $\varepsilon_t^\mu \sim N(0, \sigma_\mu^2)$. Of course, if $\sigma_\mu^2 = 0$, the specification is identical to that of Watson and Kuttner.

The output block is completed by assuming a time series process for the output gap:

$$(4) \quad z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \varepsilon_t^z,$$

where $\varepsilon_t^z \sim N(0, \sigma_z^2)$. The output gap is assumed to follow a second order autoregressive process with the two roots inside the unit circle and to have a zero unconditional mean. Note that if $z_t > 0$, actual output is *above* trend. Thus, inflationary pressures should be *positively* associated with the output gap.

Before turning to the specification of the inflation equation, it is worth noting that King and Rebelo (1993) show that in the context of this model, the IIP filter is optimal under three conditions: (i) the output gap is white noise (that is, $\phi_1 = \phi_2 = 0$), (ii) the sole shocks that affect potential output are shocks to the rate of drift (that is, $\sigma_\mu^2 = 0$), and (iii) the value of the smoothing parameter is equal to ratio of the variance of the shocks to the output gap to the variance of the shocks to the drift of potential (that is, $\sigma_z^2 / \sigma_\mu^2$). Guay and St-Amant (1996) view these conditions as unlikely to hold in practice.

2.2. Inflation

Next, we turn to the specification of the inflation equation. The leading framework for the determination of inflation is the expectations-augmented Phillips curve, a version of which can be stated as follows:

$$(5) \quad \pi_t = \lambda \pi_{t-1} + (1 - \lambda) \pi_t^e + \beta z_t + \delta(L) \varepsilon_t^\pi$$

where π_t and π_t^e denote, respectively, actual and expected inflation and $\varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$.⁸

As both the output gap and the shocks to the inflation equation are mean zero stationary processes, the rate of inflation will in steady state equal the expected inflation rate. In the short run, however, inflation can deviate from its long-run equilibrium value because of three factors. First, there is some inertia in the inflation process itself (as captured by λ), so that inflation can not instantaneously adjust to a shift in inflation expectations. This may,

⁸ Kuttner (1994) also lets the rate of inflation depend on the growth rate of actual output.

for example, be due to overlapping contracts as in Fisher (1977) or Taylor (1979). Second, deviations of output from its potential as captured by the output gap may increase the rate of inflation above its long-run expected level. The slope of the short-run Phillips curve is given by the parameter β . It should be noted that while we let inflation depend on the current output gap, Kuttner lets inflation depend on the once-lagged gap.⁹

Third, inflation may temporarily be affected by relative price changes such as a change in oil prices, the terms of trade or indirect taxes. In the basic model these other factors are captured by the disturbances, ε_t^π . The purpose of letting these obey a moving average structure is to allow the model to capture permanent shocks to the price level. Note that $\delta(1)$ measures the extent to which such shocks have permanent effects on the price level.

In order to estimate equation (5), we need to specify the process that determines expected inflation. Kuttner (1994) follows Gordon (1990), Fuhrer (1994) and many others in assuming that inflation expectations are completely backward-looking and given by last quarter's inflation rate ($\pi_t^e = \pi_{t-1}$). In such a case both demand and relative price shocks may have a permanent effect on inflation and equation (5) can be rewritten as:

$$(5) \quad \Delta\pi_t = \beta z_t + \delta(L)\varepsilon_t^\pi$$

In the absence of a good direct measure of inflation expectations for each of the G-7 countries and as it is difficult to reject a unit root for the rate of inflation in most of these countries, equation (5') seems to be a reasonable equation to estimate.^{10 11}

2.3. The methodology

The WCK methodology is based on the idea that since the output gap is unobservable, a natural way to estimate it is to use unobservable components (UC) techniques. To see how this can be done, let $Y_t = [y_t \quad \Delta\pi_t]^T$ denote the vector of observed variables, $X_t = [y_t^{pot} \quad z_t \quad z_{t-1} \quad \varepsilon_t^\pi \quad \varepsilon_{t-1}^\pi \quad \varepsilon_{t-2}^\pi \quad \varepsilon_{t-3}^\pi \quad \mu_t^y]^T$ the vector of unobserved state variables, $\mu = [0 \quad \mu^\pi]^T$ and A a vector of constants and a vector of coefficients. We then have that:

⁹ We estimated the model with Kuttner's specification, but, as evidenced by the value of the likelihood function at the optimum, found that the data preferred our specification.

¹⁰ Staiger, Stock and Watson (1996) find that their estimates of the Phillips-curve and the NAIURU are quite robust to reasonable changes in the specification of inflation expectations using either different stochastic processes for expectations or survey expectations.

¹¹ Clark, Laxton and Rose (1995a, b) use the Michigan Survey measure of inflation expectations to estimate λ directly and find that λ is about 0.4 in the United States. Fisher et al (1996) find that λ is about 0.8 in the United Kingdom. In both papers the hypothesis that the parameters on the lagged and expected inflation series are equal to unity can not be rejected.

$$(6) \quad Y_t = \mu + AX_t,$$

where:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 1 & \delta_1 & \delta_2 & \delta_3 & 0 \end{bmatrix}$$

Equation (6) is the *measurement* or *observation equation*, which states how the vector of observed variables, Y_t , is related to a vector of unobserved variables, X_t . Furthermore, define:

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

so that we can write:

$$(7) \quad X_t = BX_{t-1} + \varepsilon_t$$

where $\varepsilon_t = [\varepsilon_t^y \ \varepsilon_t^x \ 0 \ \varepsilon_t^z \ 0 \ 0 \ 0 \ \varepsilon_t^w]^T$ is a vector of disturbances, with a diagonal covariance matrix $\hat{\Omega}_\varepsilon$.

Equation (7) is the *transition equation*, which specifies the time-series process that governs the evolution of the unobservable variables. While the specification in equations (6-7) appears restricted, more complicated dynamic models can be written in this form by appropriately redefining the X_t -vector.¹²

Estimates of the model — that is, $\hat{\mu}$, \hat{A} , \hat{B} and the covariance matrix of the disturbances, $\hat{\Omega}_\varepsilon$ — can be obtained by evaluating the likelihood function using the Kalman filter. The likelihood function is given by:

¹² See Harvey (1989).

$$(8) \quad \log L = -\frac{NT}{2} \log 2\pi - \frac{1}{2} \sum_{i=1}^T \log |F_i| - \frac{1}{2} \sum_{i=1}^T v_i' F_i^{-1} v_i$$

where N , T , v_i and F_i denote the number of observed variables, the sample size, the prediction errors, and the mean square error (MSE) matrix for the prediction errors, respectively.

3. Estimated output gaps in the G-7 countries

In this section we discuss our estimates of the WCK model for the G-7 countries. The model is estimated on seasonally adjusted quarterly data of real GDP and inflation, as measured by the quarterly change in the logarithm of the consumer price index. The estimation period depends on data availability: for most countries the estimation period starts in the early 1960s and ends in 1996.¹³ To initialise the Kalman filter we follow the suggestion in Harvey (1989) and assume that X_0 is random and has a diffuse distribution, that is, its covariance matrix is given by κI , with $\kappa \rightarrow \infty$. This assumption means that nothing is known about the initial state.

We stress that the estimates reported below should be interpreted as highly preliminary. In this first draft of the paper, we wanted to see how useful the basic WCK model with minimal modifications is for the other G7 countries. As discussed below, it does surprisingly well. Nevertheless, some further country-specific fine-tuning is necessary. In a number of countries some of the parameters are insignificant, suggesting the estimation of more restricted versions of the model. On the other hand, although we are unable to reject the hypothesis that the prediction errors are serially uncorrelated in most cases, there is often evidence of non-normality of the errors. This suggests that the model specified is too restricted to fit the data. In particular, the specification of the inflation equation appears too simple. For example, it does not explicitly take into account disturbances that impact directly on the inflation rate, such as changes in value added taxes, oil price shocks, and exchange rate changes. In section 4 we discuss some of the possible extensions we hope to implement in the next draft.

With these caveats in mind, we turn to the estimates.

United States

We start with the results for the United States since it is of interest to compare our estimates with those reported by Kuttner (1994). In doing so, it should be kept in mind that Kuttner's estimation period is 1954:1-1992:4, while we use data spanning 1960:3-96:3.

¹³ For France and Italy, the estimation period start in 1970. For Germany and Italy we end in 1995. For Germany we use GDP for the pre-unification Bundesländer. All the data stem from the BIS data bank.

The parameter estimates are presented in Table 1. Note that the estimates of the autoregressive parameters for the output gap are virtually identical to those reported by Kuttner (1994, Table 6, p. 364), and are highly significant.¹⁴ Thus, ϕ_1 is estimated to be 1.6 and ϕ_2 -0.7. This implies that the responses of the output gap to shocks is "hump"-shaped. The estimate of the sensitivity of inflation to the output gap, β , is also similar to that reported by Kuttner.¹⁵ However, the second and third estimated moving average parameter for the disturbance to the inflation equation are both small and insignificant and thus quite different from Kuttner's (1994, Table 7, p. 365). The estimates suggests that about half of inflation shocks are offset in the coming quarter.

Next we turn to the estimates of potential output and the output gap. There are several ways to compute estimates of the realisation of the unobserved variables. We follow Kuttner and compute, first, an estimate of the unobserved variables given *current* data (and refer to these as "one-sided estimates"), and, second, an estimate given *all* the data ("two sided estimates"). For the last observation, these two estimates are identical by construction. However, for past output gaps, the estimates using the two-sided filter are more precise, as indicated by the narrower confidence bands, since they are based on more data. The upper row of Graph 1 contains the result using the one-sided estimates: the left panel contains a plot of actual and potential output, and the right panel contains the output gap together with plus/minus one standard error broad confidence bands. The lower row contains the analogue plots, using two-sided estimates.

Before turning to the output gaps, it is useful to briefly review the plots of actual and potential output. Given the scale, it is difficult to see much difference between the one- and two-sided estimates of potential, and we will therefore not comment on the estimates of potential output for the other countries. Note that the trend rate of growth of actual output in the United States appears relatively constant, and that recessions in the United States are followed by marked reductions in the level of output. These findings are important because, as we see below, the behaviour of output in the United States is not necessarily typical for the countries we study. In turn, this implies that the type of filters that are useful for extracting potential output is likely to vary between countries.

Next we turn to the output gaps. The two-sided estimates suggest that the United States experienced large negative output gaps in 1970-71, 1974-75, 1982-83, and 1991-92. Large positive output gaps were experienced in 1973 and 1979. In interpreting these estimates it should be recalled that the model assumes that the relationship between inflation and the output gap is stable over the estimation period. Thus, periods in which inflation

¹⁴ The standard errors are computed using what Hamilton (1994, p. 143) refers to as the *second derivative estimate* of the information matrix.

¹⁵ It should be noted that the inflation rates used in this paper are not annualised; if annualised rates were used the estimated sensitivity of inflation to the output gap would be four times larger.

changed substantially, will be characterised as having a quickly shifting output gap. This highlights the importance of controlling for direct effects on the inflation rate of different disturbances, such as tax changes and aggregate supply shocks.

It is also worth noting that at times the one-sided estimates differ quite substantially from the two-sided ones. This is a reflection of the higher uncertainty about current potential output growth if one has not yet observed actual future growth. For example, in the case of the United States the characterisation of the movements of the output gap in the second half of the 1980s is quite different whether one looks at the one-sided or the two-sided estimates. In the former case, the output gap appeared to be falling and close to zero at the end of the 1980s, whereas in the latter case, the output gap is estimated to have increased quite substantially in 1988-89.

One further reaction to the estimated output gaps is that they are rarely more than two standard errors away from zero. Thus, the output gap is typically insignificant at the five percent level. Unfortunately, since estimated output gaps are only very rarely presented together with confidence bands, it is difficult to assess whether the UC approach leads to estimates that are more or less precise than those stemming from other methodologies. Recently, Staiger, Stock and Watson (1996) have found a similar degree of imprecision for estimates of the natural rate of unemployment in a wide variety of statistical and structural models of the NAIRU.¹⁶ The extensions discussed in Section 4, may help in obtaining more precise estimates of the gap.

Finally, turning to the diagnostic tests at the bottom of Table 1, we note that while we are unable to reject the hypothesis that the prediction errors are serially uncorrelated, the Jarque-Bera test rejects the hypothesis of normality. As we see below, these findings are quite consistent across countries, which suggests that the model specified below is too restricted to fit the data. In particular, the observed skewness and outliers in the errors to the inflation equation may be due to the omission of the effects of relative price changes such as oil price and terms-of-trade shocks.

Having discussed the result for the United States, we review the results for the other G-7 countries in alphabetical order.

Canada

The estimated parameters in Canada in Table 1 are very similar to those for the United States, and we therefore do not discuss them in detail. The one parameter that does differ, however, is the estimate of the variance of shocks to the drift of potential: our

¹⁶ A typical 95% confidence interval for the NAIRU in 1990 is 5.1% to 7.7%. Using an economic criterion of smoothness, Gordon (1996) rejects the argument that the band of statistical uncertainty surrounding the NAIRU is so broad as to render the concept useless for the conduct of policy.

estimates indicate that σ_{μ}^2 is about five times larger in Canada than in the United States. This suggests that Kuttner's assumption of constant drift of potential output is too restrictive for the Canadian data.

The reason why σ_{μ}^2 is so much larger in Canada is clear from inspection of Graph 2. The plot of actual output indicates that while it grew at an approximately constant rate between 1961 and 1980, over the last fifteen years the growth rate has been declining slowly over time. This shift is accounted for by a gradual decline of μ_t^y . The two-sided estimates of the output gap indicate that the Canadian economy has experienced three large recessions in the sample period: in 1982-83, when the output gap was larger than four percent, in 1974-75 and in 1991-92, when the output gaps were about two percent.

France, Germany and Italy

The parameter estimates for France are quite similar to those for the United States and Canada, and it therefore is not necessary to go through them in detail. The main difference is that the sensitivity of inflation with respect to the output gap is insignificant (asymptotic t-value = 1.3). The estimated output gaps were negative in the 1983-86 and the 1993-94 period.

Turning to the results for Germany, two findings are of interest. First, the estimated autoregressive coefficients differ from those in the other countries. While ϕ_1 typically is estimated to be about 1.5 and ϕ_2 around -0.7, in Germany ϕ_1 is estimated to be close to 0.7, and ϕ_2 close to 0.1 (and insignificant). This implies that the estimated output gap in Germany does not display a hump-shaped reaction to ε_t^r -shocks. Note also that δ_1 is close to minus unity in Germany. This suggests that the ε_t^r -shocks may be thought of as permanent shocks to the price level, which have a temporary impact on the rate of inflation. The estimates of the realisation of the output gap in Germany appear much less smooth than those for the other countries. This may be due to the fact that there is still some seasonality in the German data. We are currently exploring ways to deal with this problem. The two-sided estimates of the output gaps show that the output gap became increasingly positive following German unification in 1989. From 1991 until 1993, however, the gap fell. While it appears to have become smaller since 1993, most recently the data suggests that it has widened somewhat.

The parameter estimates for Italy are more similar to those for Canada and the United States than to those of Germany. Two differences are noteworthy. First, the second- and third-order MA coefficients are of opposite sign, but numerically similar and highly significant. This suggests a rather complex reaction of inflation to ε_t^r -shocks. Second, the variance of the disturbances in the inflation equation is about three times larger in Italy than in the other countries discussed so far. The estimated output gap is currently positive.

Japan

Plots of actual output in Graph 6 suggests that the behavior of output is quite different in Japan compared with in the other countries. With the exception of the period 1973-75, it is in Japan very rare for actual output to fall. Indeed, the Japanese output data is heavily influenced by three growth phases: a high growth phase ending in 1973, a moderate growth phase between 1973 and 1990, and a low growth phase thereafter.

Despite this, the estimates of the autoregressive parameters for the Japanese output gap are quite similar to those of the other countries. In contrast, the variance of the shocks to the drift of potential output, σ_u^2 , is much higher than in the other countries, while the variance of the shocks to the output gap, σ_z^2 , is relatively low. The sensitivity of inflation to the output gap, β , is also much higher than in the other countries. Thus, the Japanese economy tends to experience relatively large shocks to the growth rate of potential output, and small variations in the output gap. However, these small movements in the output gap elicit relatively large movements in inflation.

Turning to the estimates of the output gap in Graph 6, the model suggests that the gap has been essentially zero since the early 1980s. Before then, however, Japan apparently experienced cyclical fluctuations of the same magnitude as the other countries under study.

United Kingdom

Finally we consider the estimates for the United Kingdom. The results in Table 1 are similar to those of the other countries. There are three exceptions. First, β is larger than in the other countries (except) Japan. Thus, movements in the output gap has comparatively large inflation effects in the United Kingdom. Second, the variance of the shocks to the drift of potential output is zero. The model thus suggests that the rate of drift of potential was constant in the United Kingdom in the estimation period. Third, the variance of the shocks to the inflation equation are quite large.

The plot of actual output in Graph 7 suggests that the behavior of output in the United Kingdom changed in the late 1970s: in the first half of the sample output grew at a relatively constant rate. In the early 1980s and in the early 1990s, however, there were two episodes during which output fell for several quarters. The episodes, together with the recession following the first oil shock, stand out clearly in the two-sided estimates of the output gap.

4. Extensions [to be completed]

The results discussed in the previous section illustrate that the basic WCK model can be usefully applied to most of the G-7 countries. In this section we discuss a number of potential criticisms and possible extensions. The two following subsections focus in turn on the output and the inflation equations.

4.1. Output

The model for output in equations (1) to (4) is very similar to the univariate model of Watson (1986) and Clark (1989). This keeps it relatively simple to estimate at the cost of possibly imposing implausible restrictions. One of these is that the potential output process follows a random walk model. St-Amant and van Norden (1996) argue that restricting the dynamics of the response of output to potential output shocks to a random-walk model may not be appropriate. Indeed, standard real business cycle models suggest that technology shocks are absorbed gradually by the economy. Adjustment costs for capital and labour, learning and diffusion processes, habit formation, and time to build imply richer dynamics than a random walk for these shocks. Results from applying methodologies such as the structural VARs of Blanchard and Quah (1989), Shapiro and Watson (1989) and King et al. (1991) suggest that permanent shocks feed only gradually into output. However, whether this is due to factors such as time to build, or whether it is due to the fact that demand does not immediately adjust to increased supply, is difficult to assess. The finding of those studies that inflation tends to *fall* in response to permanent output shocks suggests that the latter is a more likely explanation, in which case it may make more sense to just consider the long-run effects of these shocks as potential output shocks.¹⁷

A second implicit restriction in the output model is that there is no feedback from inflation to the output gap. If central banks respond to a rise in inflation by tightening monetary conditions and raising real interest rates, such a rise may have a subsequent negative impact on the output gap. In the next draft we intend to allow for such feedback effects.

4.2. Inflation

Also the inflation equation is kept very simple. First, in the basic model we followed Kuttner (1994) and others by assuming that inflation expectations are completely backward-looking and given by last period's inflation. While this assumption is not unreasonable, it would be useful to check the robustness of the results with respect to different specifications of the inflation expectations process.

¹⁷ Implicitly this is the rationale behind the decomposition proposed by Beveridge and Nelson (1981).

One modification is suggested by the fact that in some of the other G-7 countries, in particular Germany, rejections of the non-stationarity of inflation are more frequent. The smaller degree of persistence in inflation could be a result of the fact that over the sample period inflation expectations put some weight on the central bank's fixed inflation target. Assume, for example, that inflation expectations are a weighted average of past inflation and the central bank's constant inflation target:

$$(9) \quad \pi_t^e = \rho\pi_{t-1} + (1-\rho)\pi^*$$

where $0 \leq \rho \leq 1$. In this case equation (5) becomes:

$$(5'') \quad \pi_t = \theta_0 + \theta_1\pi_{t-1} + \beta z_t + \delta(L)\varepsilon_t^\pi$$

where $\theta_0 = (1-\lambda)(1-\rho)\pi^*$ and $\theta_1 = 1 + \lambda(1-\rho)$.

However, replacing equation (5') by (5'') in the case of Germany did not materially affect the results reported in Table 1.¹⁸

Second, other shocks and in particular relative price shocks may affect inflation. In a relatively closed economy such as the United States, Kuttner's specification of the inflation equation as a Phillips curve where the change in inflation is primarily affected by the output gap may be appropriate. However, the other G-7 economies are more open, so that exchange rate and oil price changes may have non-negligible effects on inflation. We can allow for such effects by adding additional regressors in the inflation equation (5).

A more general specification would also let the acceleration in inflation depend on both the current and the lagged gap. An extended version of the Phillips curve equation can then be written as:

$$(5''') \quad \Delta \pi_t = \beta_0 z_t + \beta_1 z_{t-1} + \gamma(L)\omega_t + \delta(L)\varepsilon_t^\pi$$

where the vector ω , captures other factors such as changes in oil prices or exchange rates that may temporarily affect inflation. Equation (5''') permits us to test whether inflation depends on the level, or the change in the output gap.

Table 2 and Graph 8 illustrate the consequences of estimating a more general inflation equation in the case of Canada. The estimated inflation equation is of the form:

$$(10) \quad \Delta \pi_t = \beta_0 z_t + \beta_1 z_{t-1} + \delta(L)\varepsilon_t^\pi + \mu_\pi + \gamma_0 de_t + \gamma_1 de_{t-1} + \gamma_2 de_{t-2} + \rho_0 do_t + \rho_1 do_{t-1} + \rho_2 do_{t-2}$$

where de_t and do_t are log changes in the nominal trade-weighted exchange rate and oil prices at time t .

¹⁸ In the absence of survey measures of inflation expectations, an alternative strategy could be to use nominal interest rates to measure inflation expectations. See Debelle and Laxton (1996).

In comparing the two sets of estimates it should be kept in mind that the sample periods are somewhat different. Table 2 and Graph 8 indicate that the estimated process for the output gap is quite similar. However, the results also suggest that it is indeed important to consider more general specifications of the inflation equation. First, we find significant effects on inflation of both changes in the exchange rate and oil prices. A 1 percent appreciation of the trade-weighted exchange rate leads to a contemporaneous fall in annualised inflation of about 20 basis points. Similarly, a 1 percent increase in energy prices leads to a significant rise in annualised inflation of about 5 basis points. The inclusion of these relative price shocks appears to have an effect both on the precision of the estimates and on some of the short-run movements of the output gap. One particular example is the effect of the fall in oil prices in 1986-87. The inclusion of the oil price variable implies that the output gap in the late 1980s is now estimated to be significantly positive.

Second, the parameter on the current output gap is much larger and the parameter on the lagged output gap is negative and significantly different from zero, suggesting substantial acceleration effects. The estimates suggest that a 1 percentage point change in the output gap leads to a contemporaneous rise in annualised inflation of about 60 basis points.

These results suggest that it is worth investigating the impact of additional supply shocks on inflation also for the other countries. These will be incorporated in the next draft of this paper.

5. Conclusions

Because of its central role in the determination of wages and prices, the output gap plays an important role in the conduct of monetary policy. However, because data on potential output is not directly observable, policy must be conducted on the basis of estimates of the output gap. In this paper we show that a relatively simple model based on Watson (1986), Clark (1989) and Kuttner (1994) estimated using unobservable component techniques can be usefully applied to most of the G7 countries. With the exception of Germany, we find that in all countries the stochastic process of the output gap is well captured by an AR(2) model with the first AR parameter significantly larger than one and the second significantly negative, suggesting that the typical response of the output gap to a shock is hump-shaped. In all countries, the slope of the short-run Phillips curve is positive, and, with the exception of France, significantly different from zero.

However, using the basic Watson-Clark-Kuttner model we find that the statistical confidence bands around the output gap are typically quite large. This suggests that using additional information to increase the precision of the estimates is necessary. For example, in the case of Canada we find that the fit of the model is indeed improved when we add additional relative price variables in the inflation equation and allow for acceleration

--- Preliminary and incomplete draft ---

effects of the output gap on inflation. This suggest the need for some further fine-tuning of our basic results.

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Table 1

Maximum likelihood estimates				
Country and sample period	Canada 1961:2 - 1996:3	France 1970:2 - 1996:3	Germany 1960:3 - 1995:4	Italy 1970:2 - 1995:4
ϕ_1	1.566 (0.061) [25.52]	1.540 (0.113) [13.68]	0.675 (0.075) [9.001]	1.489 (0.073) [20.28]
ϕ_2	-0.675 (0.056) [12.15]	-0.603 (0.097) [6.227]	0.127 (0.073) [1.746]	-0.650 (0.068) [9.623]
β	0.062 (0.016) [3.765]	0.041 (0.032) [1.291]	0.058 (0.016) [3.600]	0.065 (0.031) [2.081]
δ_1	-0.864 (0.079) [10.92]	-0.626 (0.114) [5.519]	-1.070 (0.081) [13.23]	-0.742 (0.085) [8.699]
δ_2	-0.061 (0.078) [0.776]	0.154 (0.119) [1.298]	0.124 (0.111) [1.119]	0.482 (0.082) [5.903]
δ_3	-0.046 (0.077) [0.603]	-0.126 (0.094) [1.342]	0.054 (0.097) [0.554]	-0.398 (0.074) [5.359]
μ^π	0.001 (0.019) [0.029]	-0.012 (0.028) [0.445]	-0.002 (0.017) [0.138]	-0.002 (0.755) [0.024]
σ_y^2	0.559	0.252	0.901	0.099
$\sigma_\mu^2 \times 100$	0.470	0.140	0.187	0.336
σ_z^2	0.137	0.102	0.053	0.318
σ_π^2	0.184	0.224	0.173	0.656
Log-likelihood	-159.03	-81.10	-196.91	-145.35
Diagnostic tests				
<i>Output eq.</i>				
Q-Stat. ^a	0.963	0.724	0.374	0.227
J-B ^b	0.309	0.002	0.073	0.861
<i>Inflation eq.</i>				
Q-Stat.	0.414	0.173	0.626	0.023
J-B	0.199	0.000	0.299	0.000

Notes: Standard errors in parenthesis and t-statistics in brackets.

^a p-value for Q-statistic for a test of the hypothesis of no fourth-order serial correlation of the prediction errors.

^b p-value for Jarque-Bera test for normality.

Table 1 (continued)

Maximum likelihood estimates			
Country and sample period	Japan 1960:3 - 1996:3	United Kingdom 1960.3 - 1996:3	United States 1960:3 - 1996:3
ϕ_1	1.633 (0.046) [35.55]	1.817 (0.044) [41.50]	1.580 (0.103) [15.32]
ϕ_2	-0.851 (0.044) [19.39]	-0.906 (0.042) [21.75]	-0.683 (0.099) [6.918]
β	0.266 (0.043) [6.211]	0.130 (0.031) [4.235]	0.050 (0.022) [2.241]
δ_1	-1.109 (0.073) [15.25]	-0.777 (0.061) [12.80]	-0.694 (0.100) [6.937]
δ_2	-0.025 (0.075) [0.333]	0.029 (0.061) [0.484]	0.077 (0.116) [0.666]
δ_3	0.151 (0.071) [2.118]	-0.206 (0.061) [3.393]	0.132 (0.097) [1.371]
μ^π	-0.012 (0.020) [0.578]	-0.001 (0.061) [0.023]	0.009 (0.002) [0.366]
σ_y^2	0.641	1.040	0.456
$\sigma_\mu^2 \times 100$	1.936	0.000	0.087
σ_z^2	0.038	0.025	0.193
σ_π^2	0.560	0.639	0.218
Log-likelihood	-257.35	-271.40	-154.45
Diagnostic tests			
<i>Output eq.</i>			
Q-Stat. ^a	0.558	0.236	0.740
J-B ^b	0.000	0.000	0.002
<i>Inflation eq.</i>			
Q-Stat.	0.050	0.395	0.401
J-B	0.000	0.000	0.000

Notes: Standard errors in parenthesis and t-statistics in brackets.

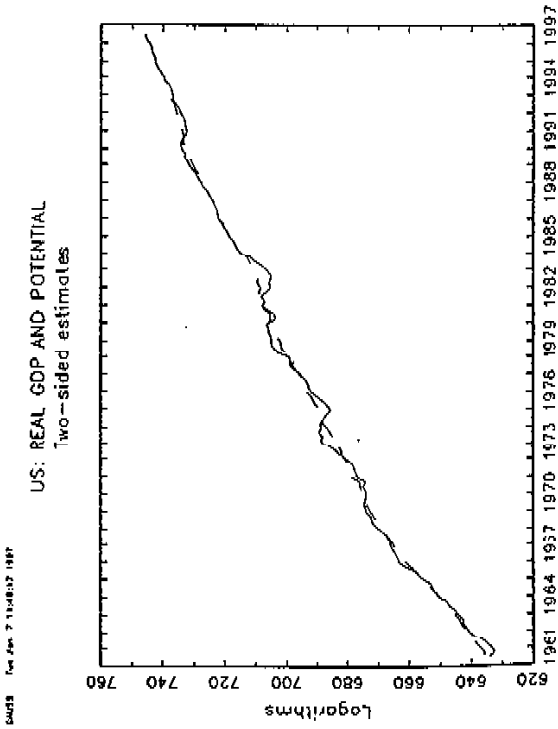
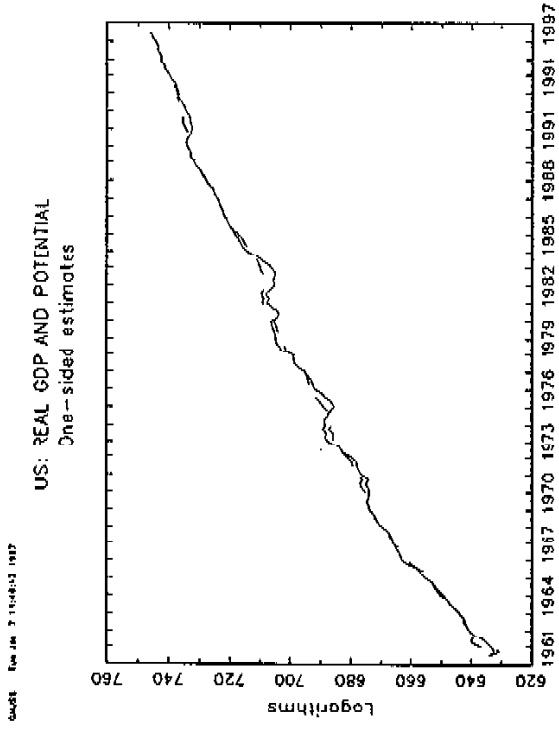
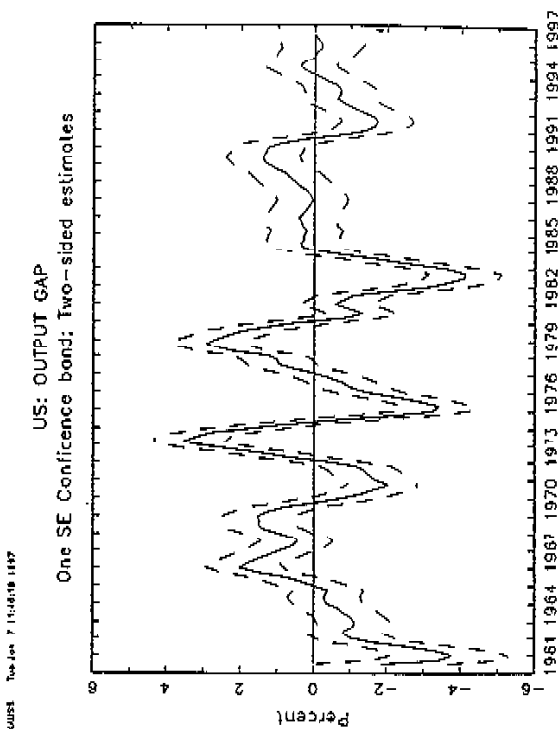
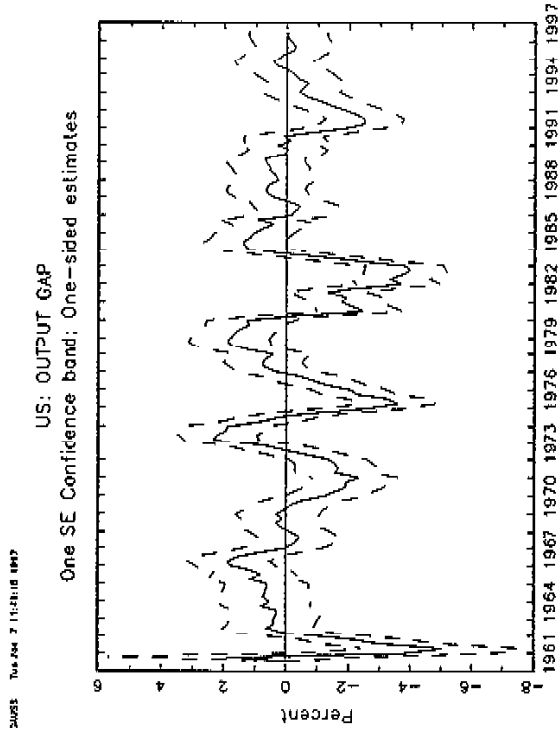
^a p-value for Q-statistic for a test of the hypothesis of no fourth-order serial correlation of the prediction errors.

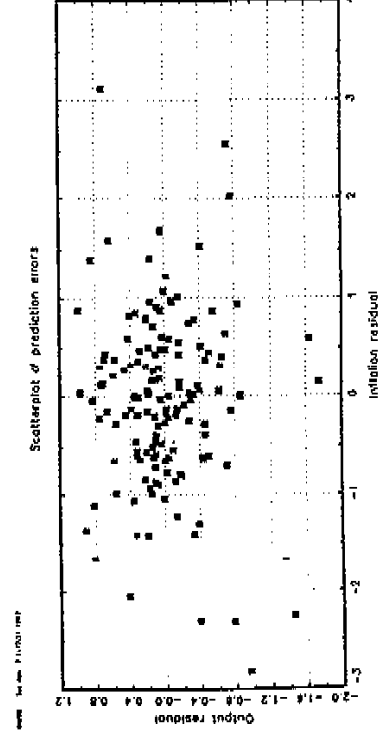
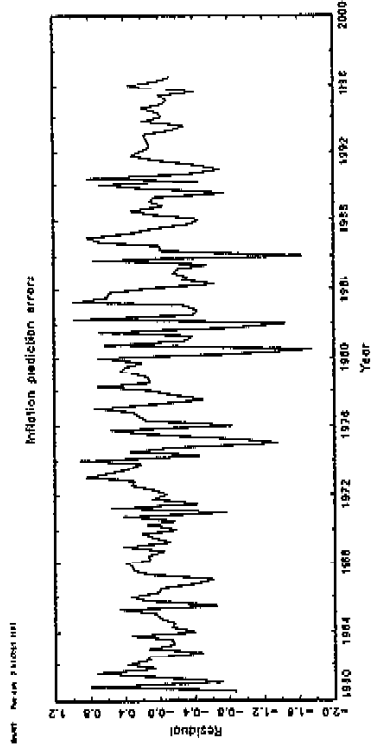
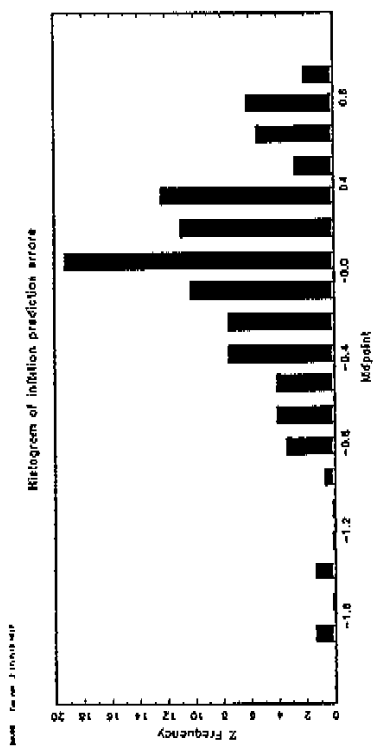
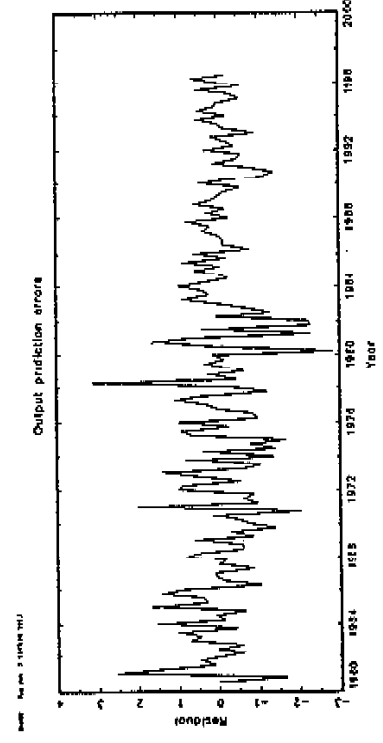
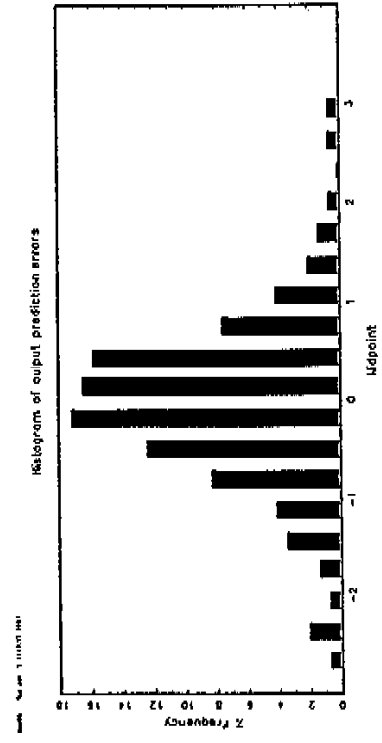
^b p-value for Jarque-Bera test for normality.

Table 2
The basic and extended model (Canada)

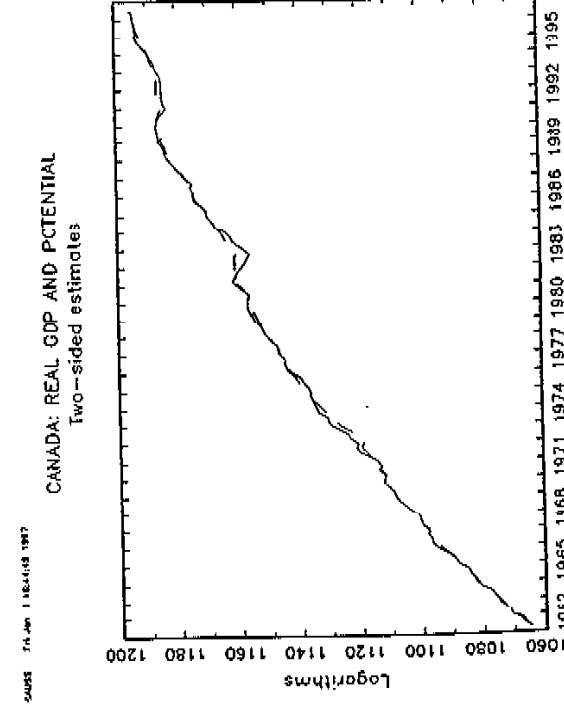
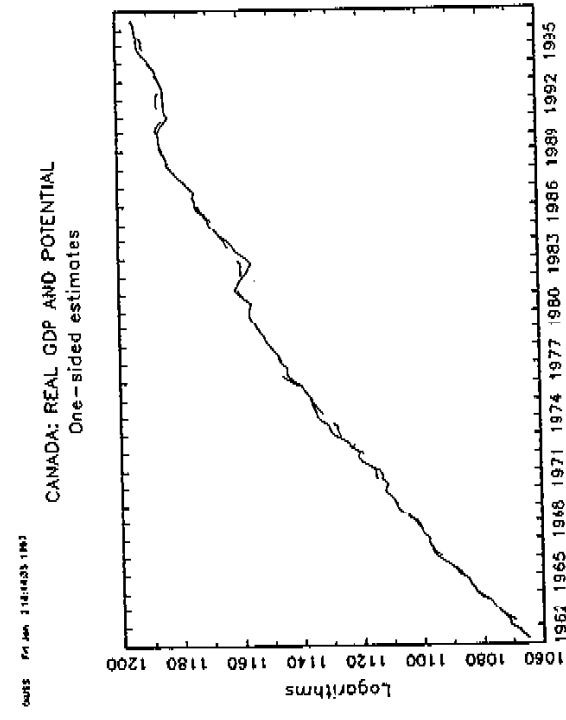
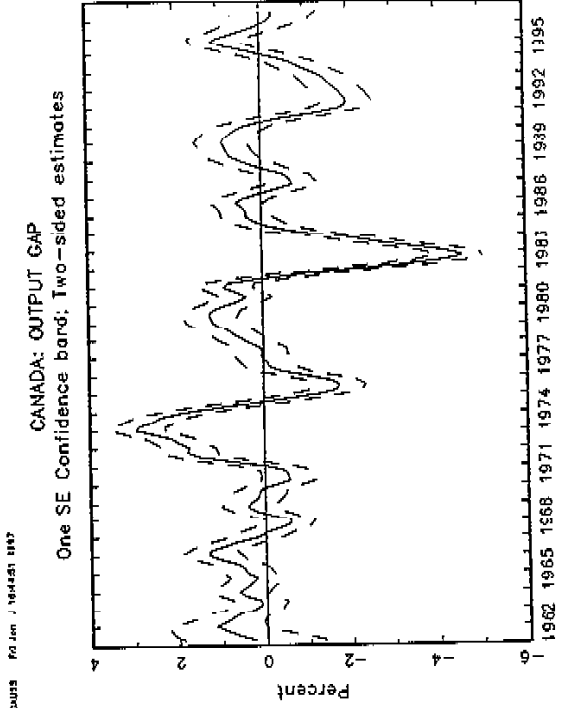
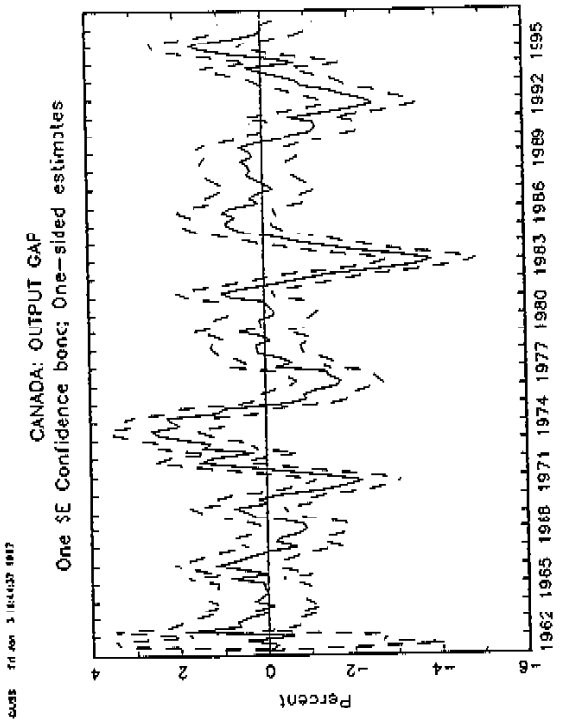
Maximum likelihood estimates				
Country and sample period	Canada 1961:2 - 1996:3	Canada 1964:1 - 1996:3		
ϕ_1	1.566 (0.061) [25.52]	1.65 (0.17) [9.49]		
ϕ_2	-0.675 (0.056) [12.15]	-0.73 (0.15) [4.65]		
β_0	0.062 (0.016) [3.765]	0.15 (0.05) [2.95]		
β_1		-0.12 (0.05) [2.26]		
δ_1	-0.864 (0.079) [10.92]	-0.90 (0.23) [3.87]		
δ_2	-0.061 (0.078) [0.776]	-0.01 (0.01) [0.15]		
δ_3	-0.046 (0.077) [0.603]	-0.00 (0.09) [0.05]		
γ_0	-	-0.05 (0.02) [1.69]		
γ_1	-	0.01 (0.04) [0.22]		
γ_2	-	0.03 (0.03) [1.22]		
ρ_0	-	0.01 (0.00) [3.34]		
ρ_1	-	-0.00 (0.00) [1.24]		
ρ_2	-	-0.00 (0.00) [0.29]		
μ_π	0.001 (0.019) [0.029]	-0.016 (0.015) [1.10]		
σ_y^2	0.559	0.519		
σ_z^2	0.137	0.138		
σ_π^2	0.184	0.179		

GRAPH 4

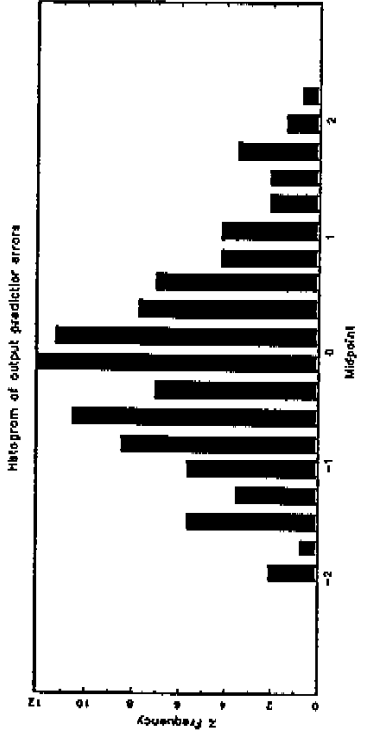




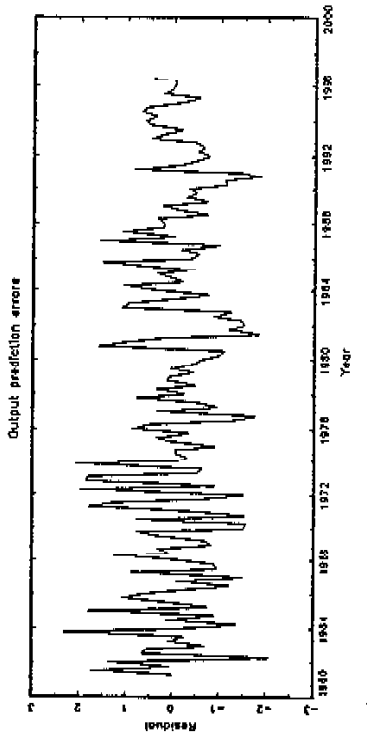
GRAPH 2



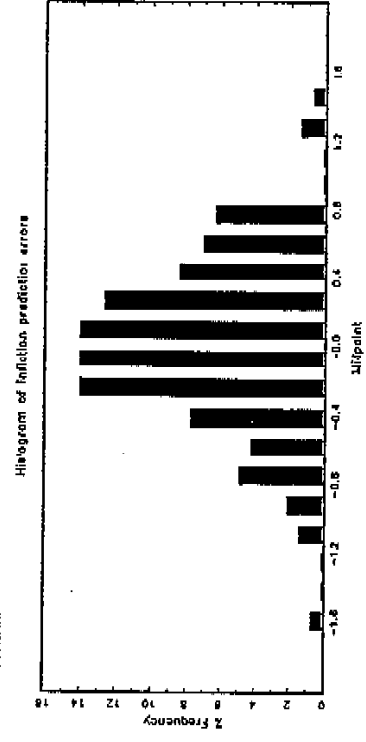
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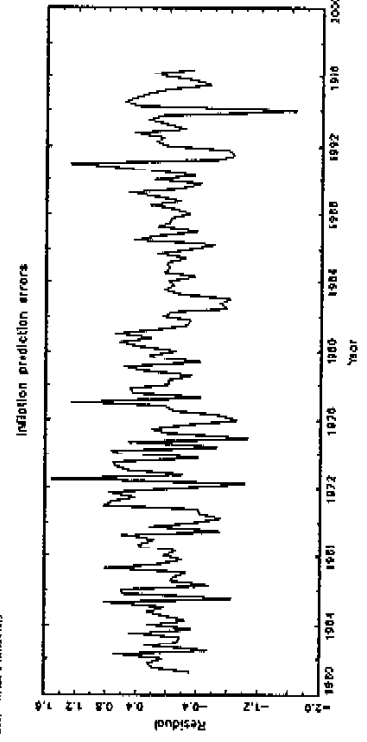
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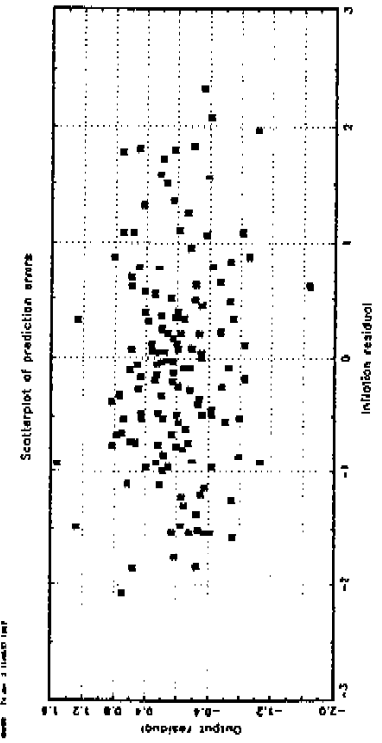
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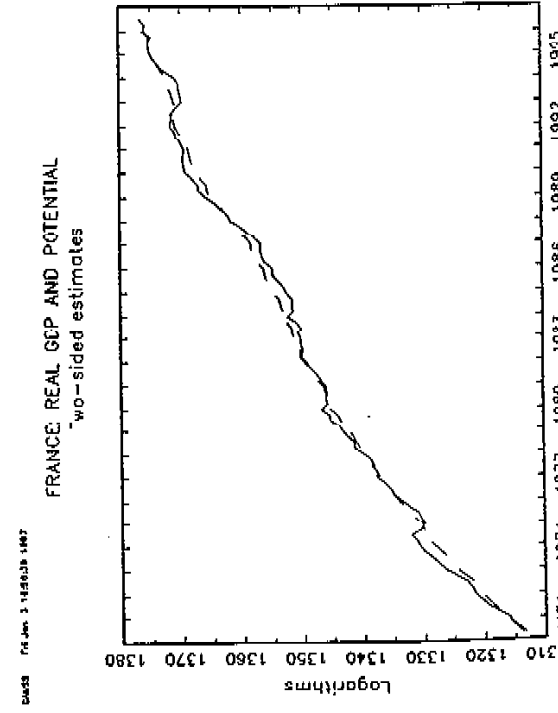
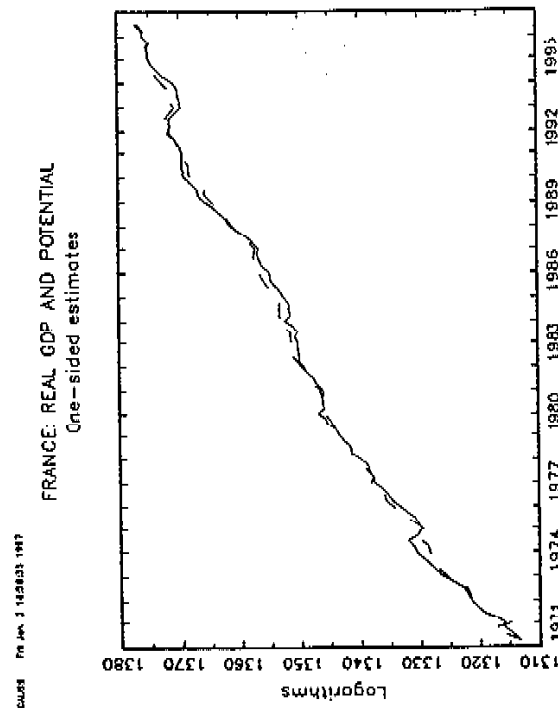
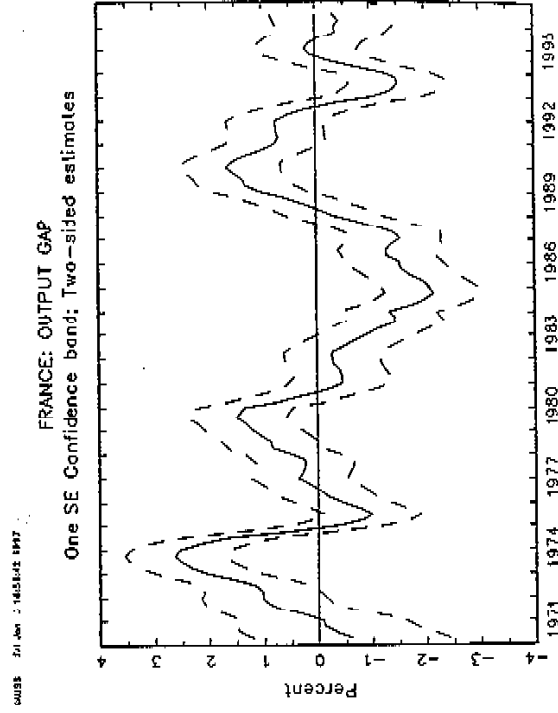
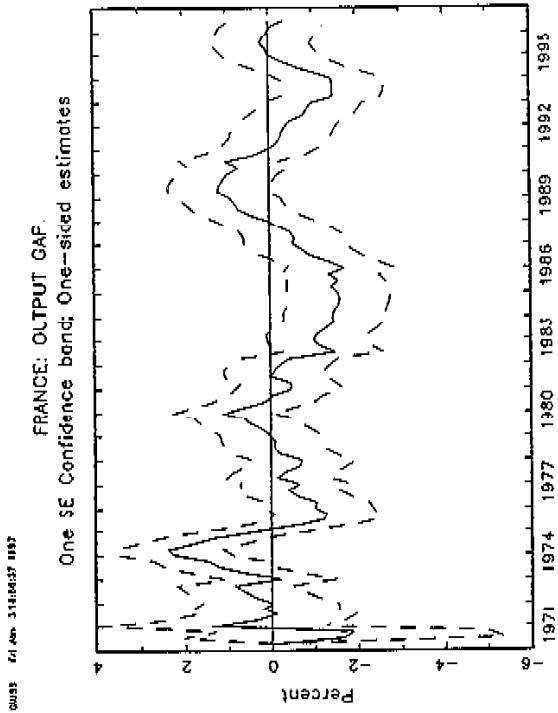
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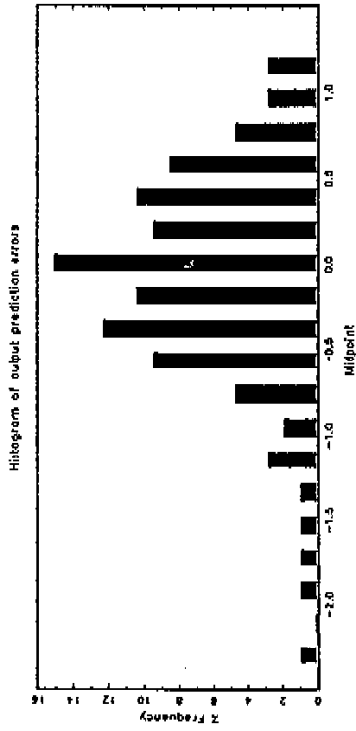
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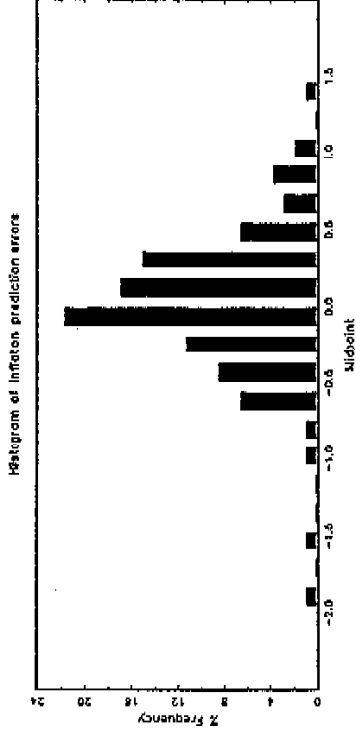
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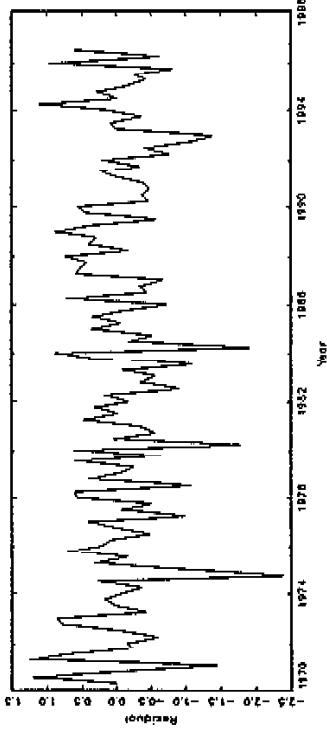
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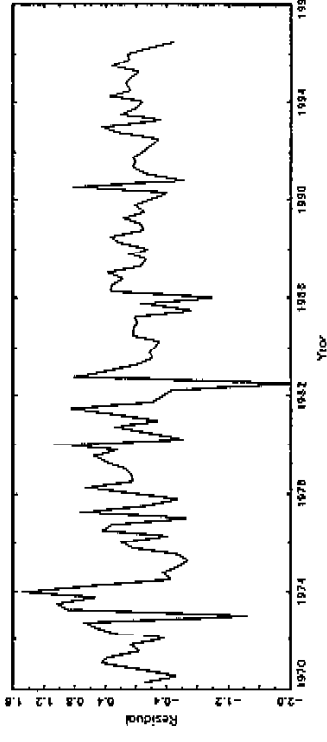
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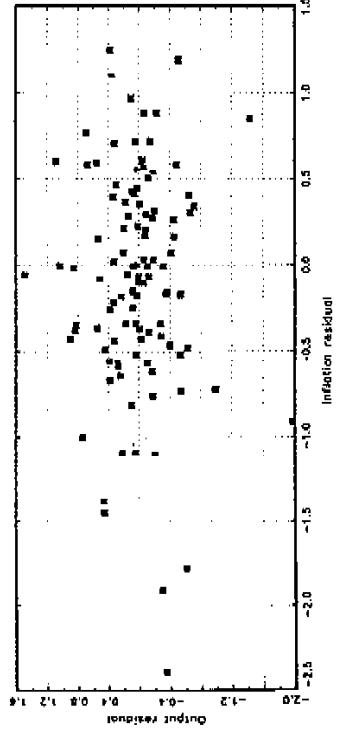
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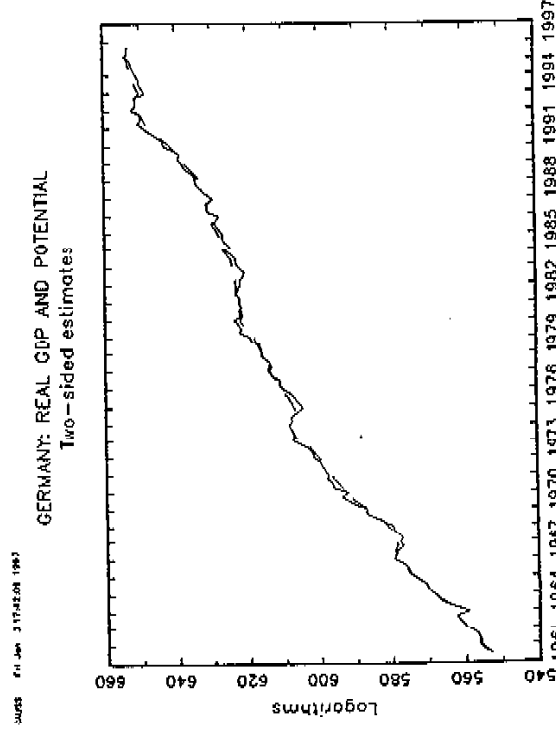
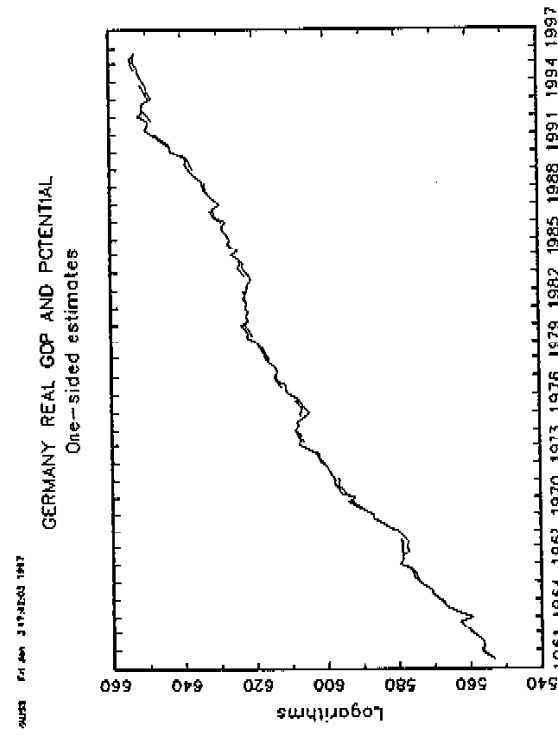
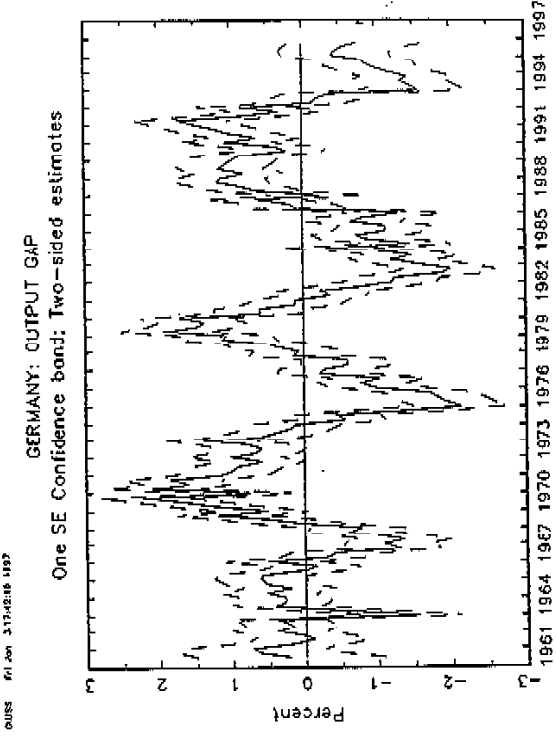
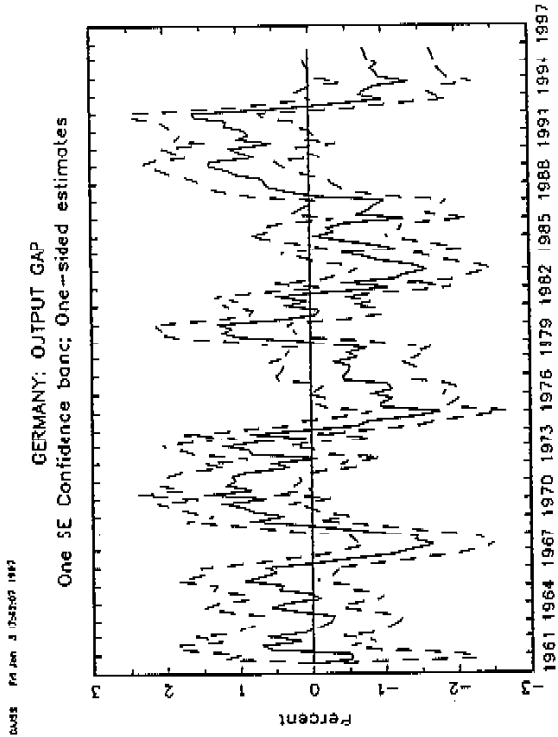
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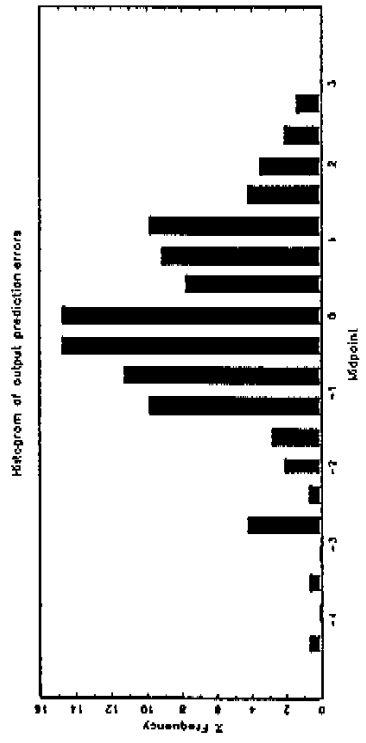
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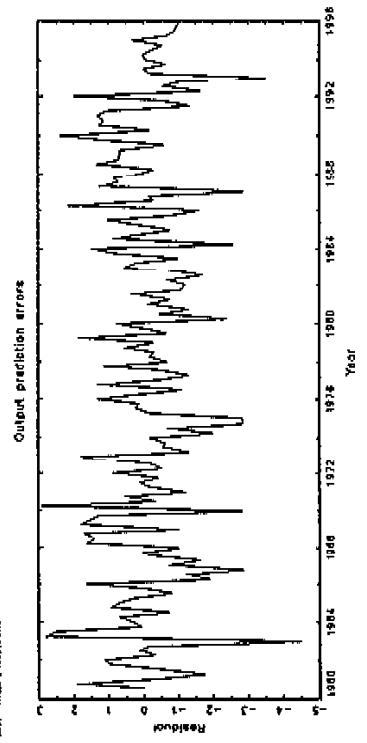
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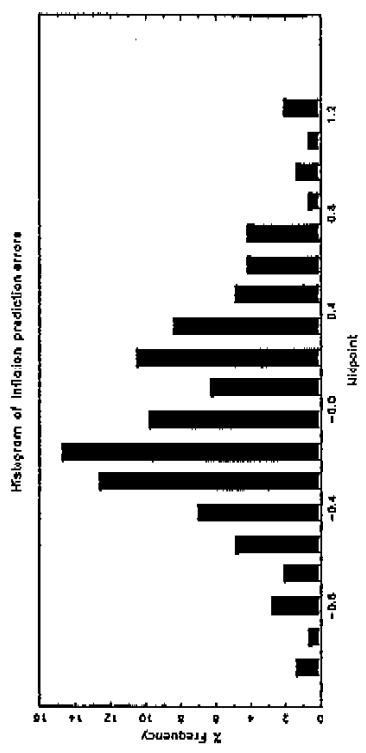
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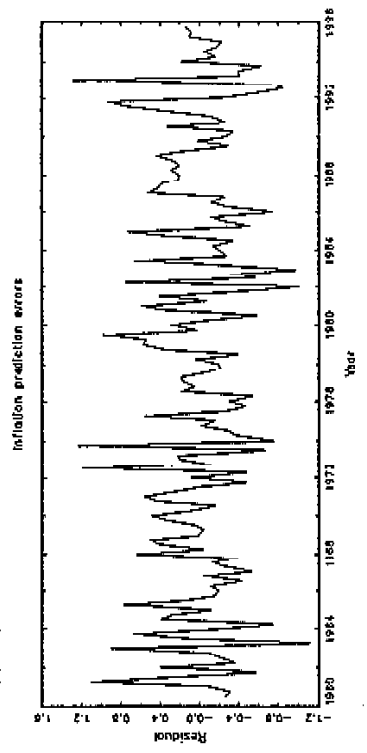
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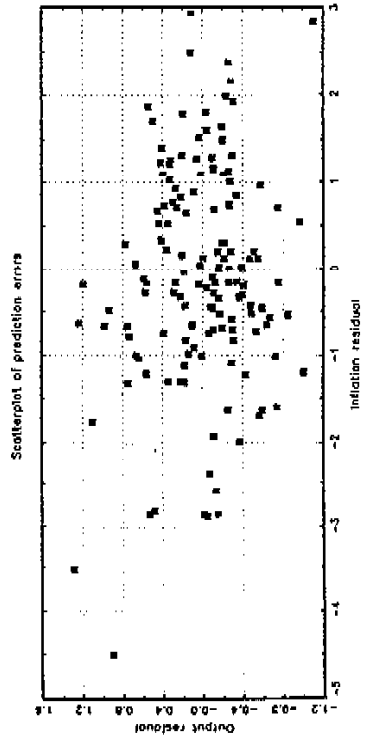
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DATE: 1996.01.15 10:00:00



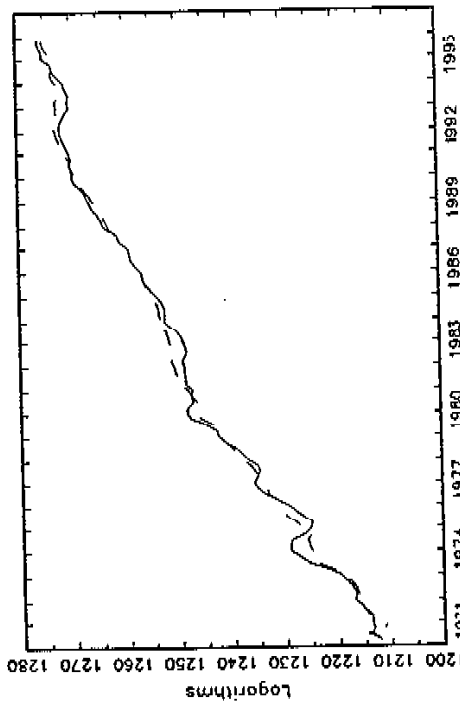
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GRAPH 5

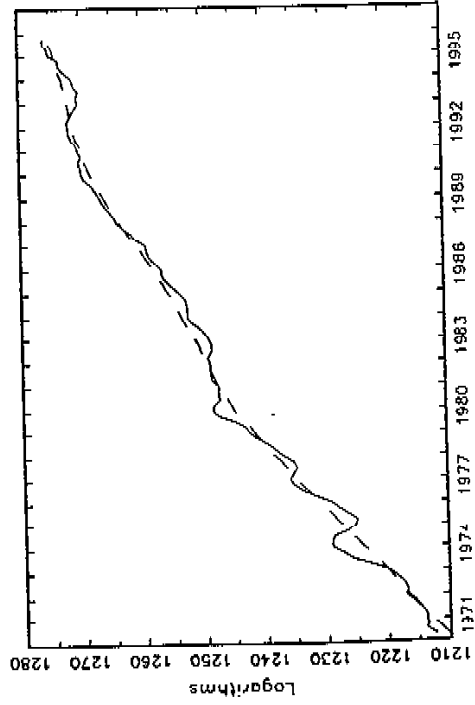
CAUSE Fri Jan 3 15:01:50 1997

ITALY: REAL GDP AND POTENTIAL
One-sided estimates



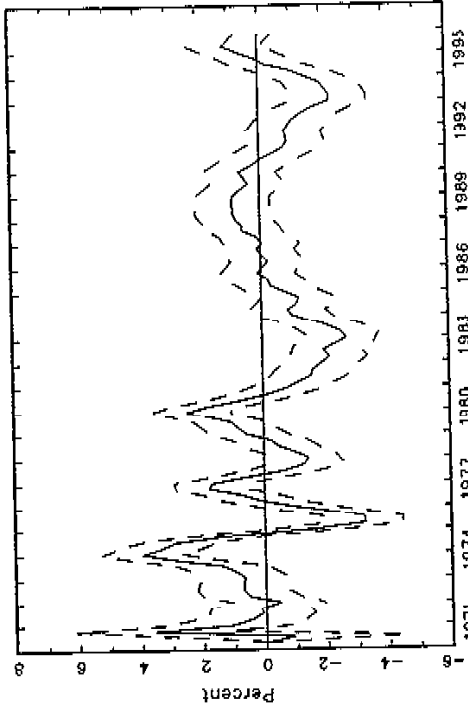
CAUSE Fri Jan 3 15:01:54 1997

ITALY: REAL GDP AND POTENTIAL
Two-sided estimates



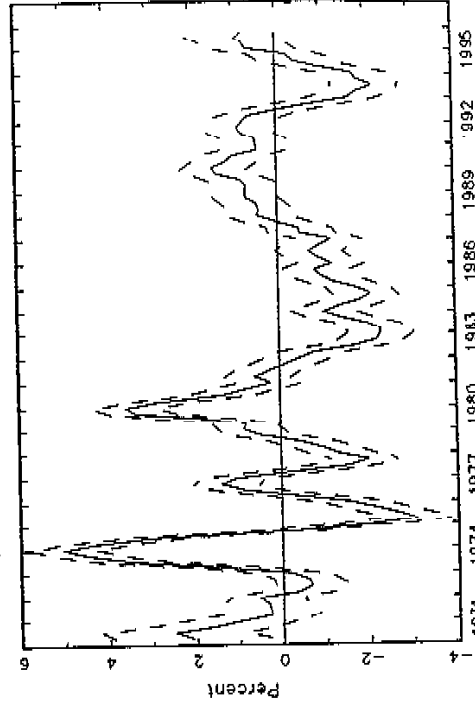
CAUSE Fri Jan 3 15:01:53 1997

ITALY: OUTPUT GAP
One SE Confidence band; One-sided estimates

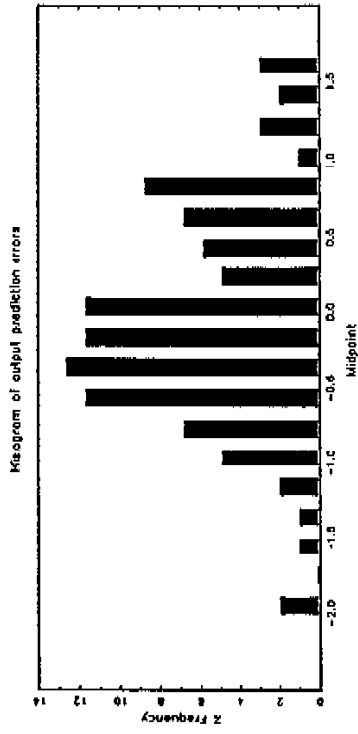


CAUSE Fri Jan 3 15:01:58 1997

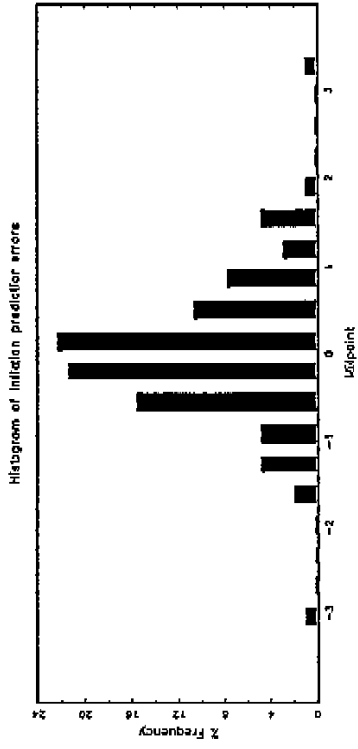
ITALY: OUTPUT GAP
One SE Confidence band; Two-sided estimates



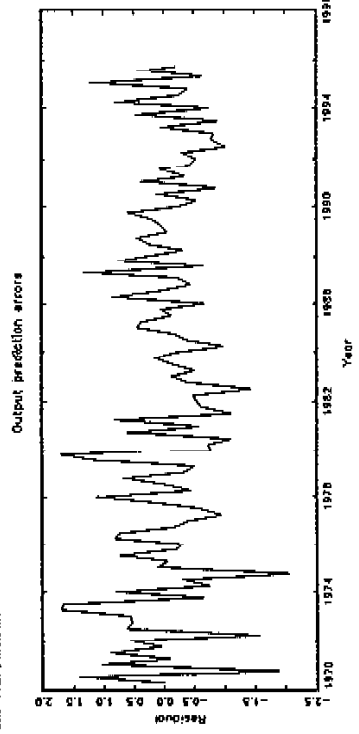
Source: Penn State Harrisburg



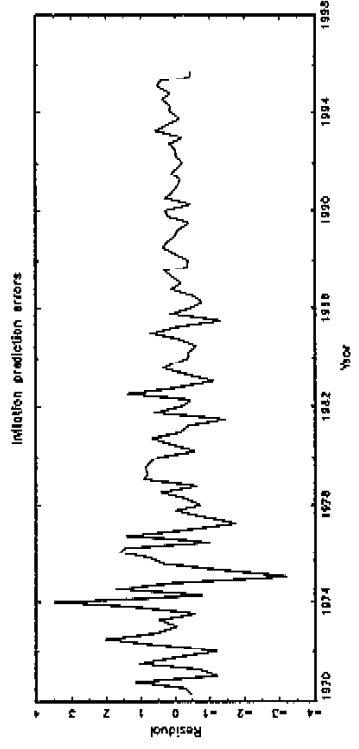
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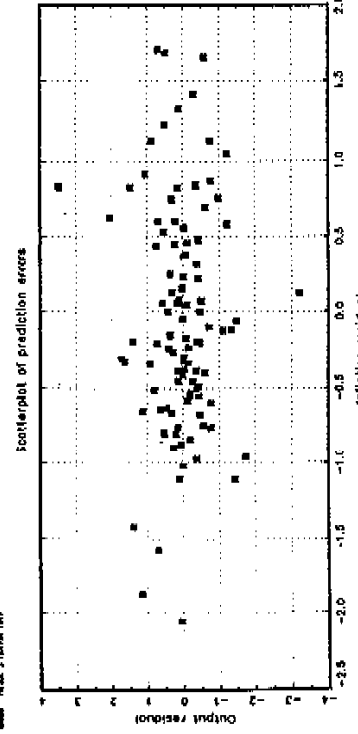
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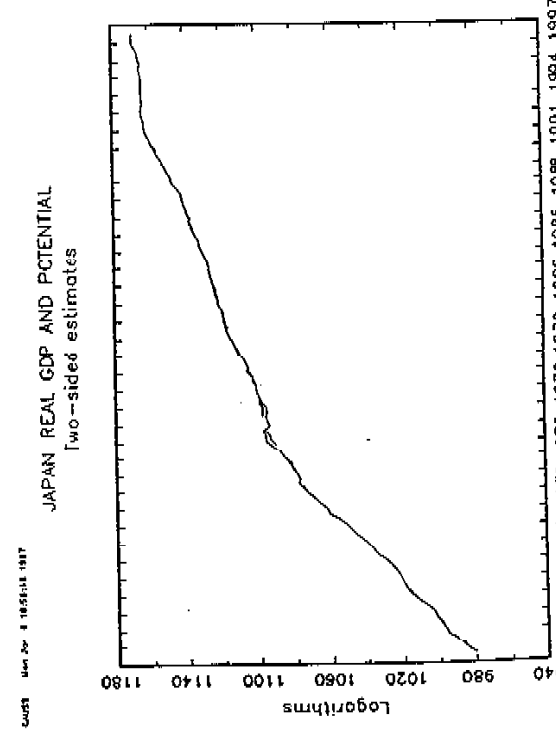
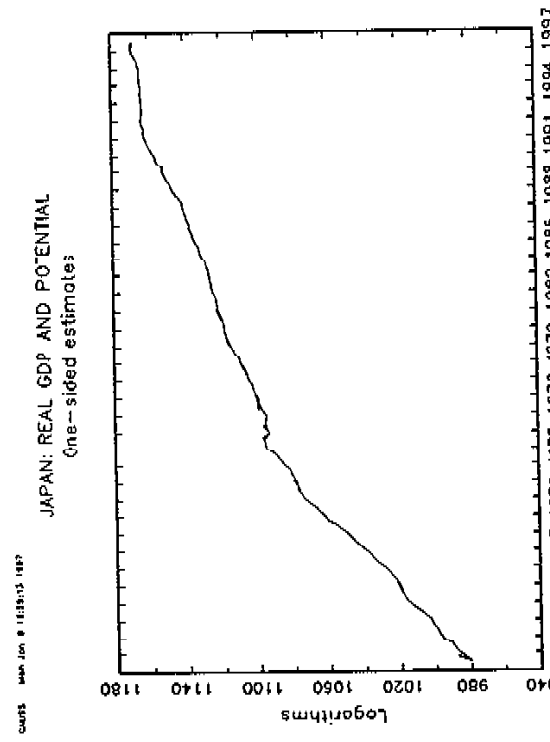
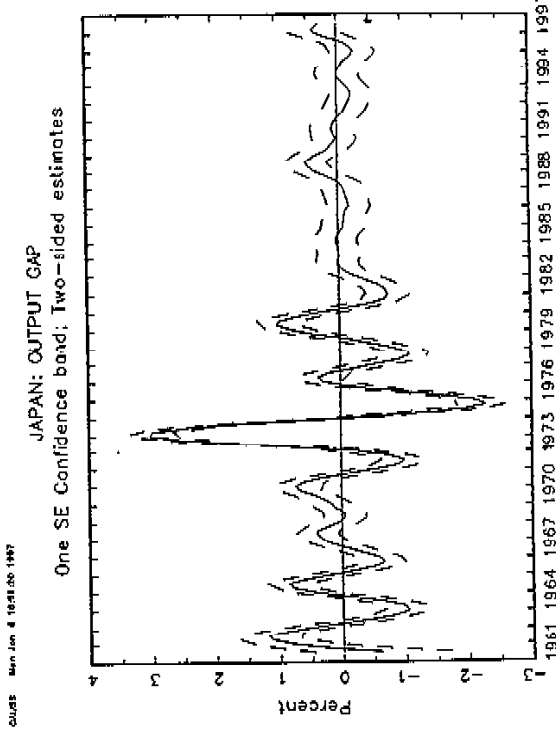
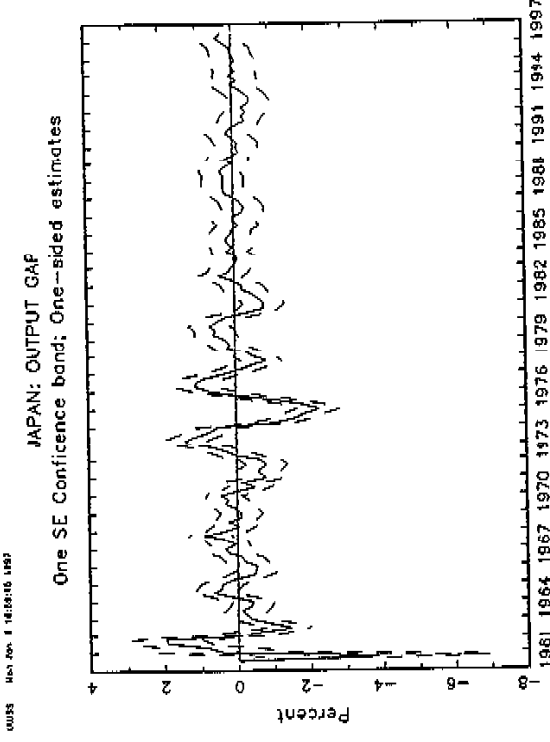
Source: Penn State Harrisburg



Source: Penn State Harrisburg

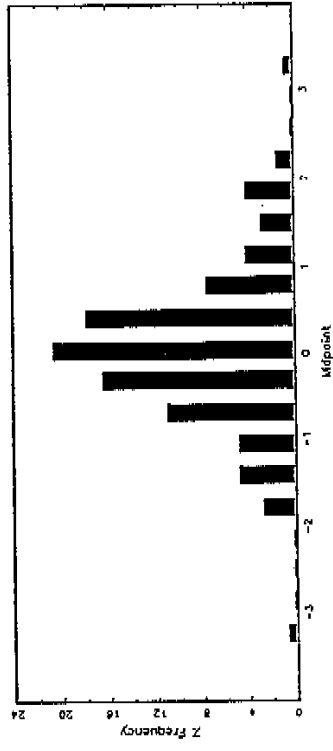


GRAPH 6



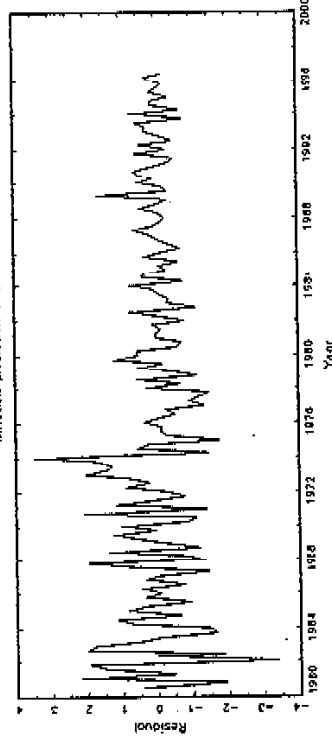
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Histogram of inflation prediction errors



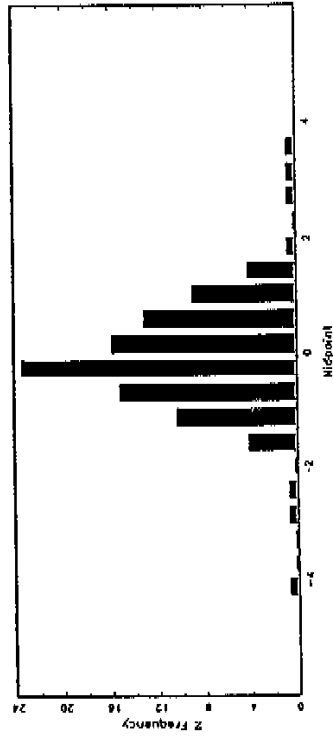
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Inflation prediction errors



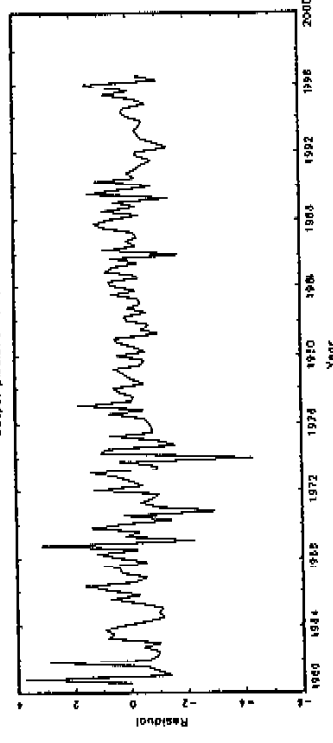
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Histogram of output prediction errors



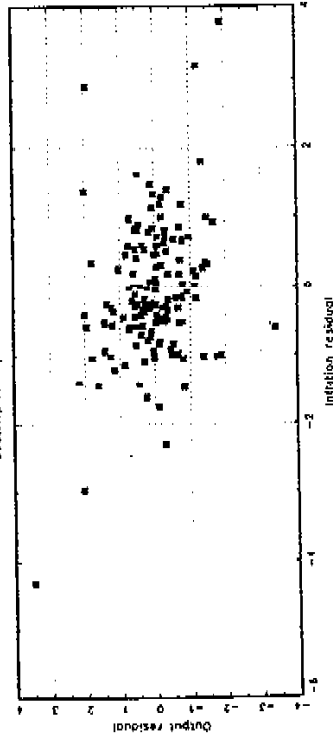
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Output prediction errors



DATE: 1999.04.14 10:21:17 AM

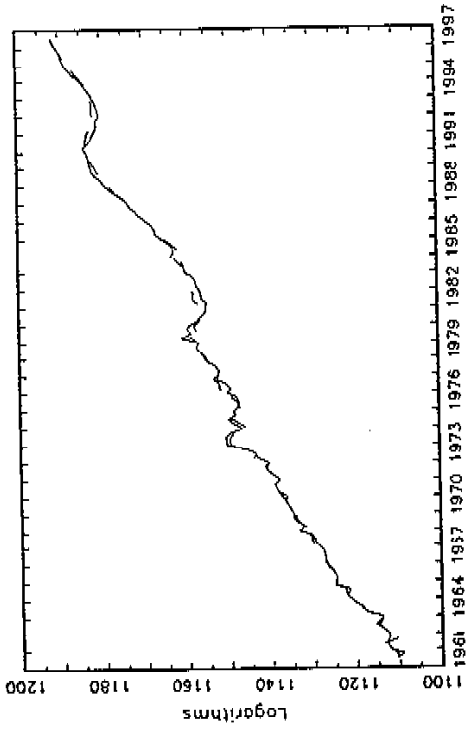
Scatterplot of prediction errors



GRAPH 7

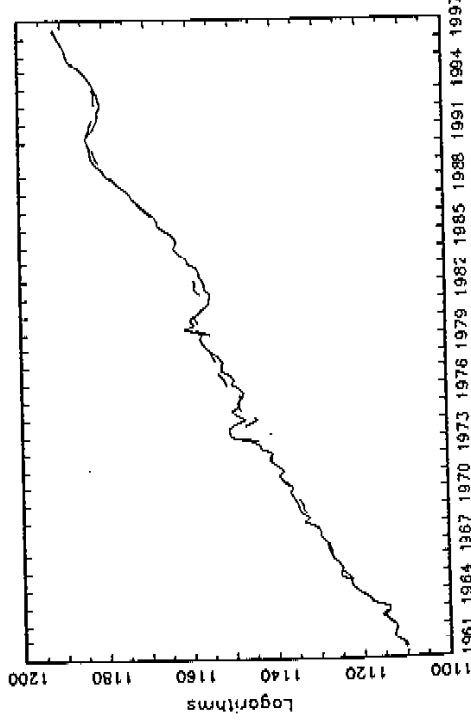
DAUSE Fri Aug 3 16:16:28 1997

UK: REAL GDP AND POTENTIAL
One-sided estimates



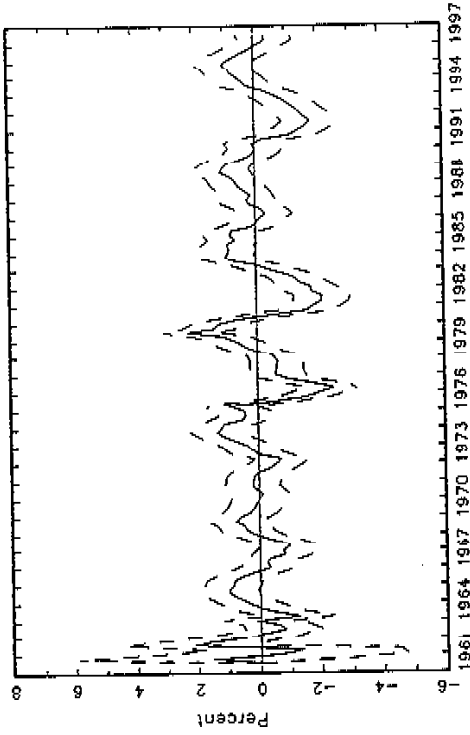
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UK: REAL GDP AND POTENTIAL
Two-sided estimates



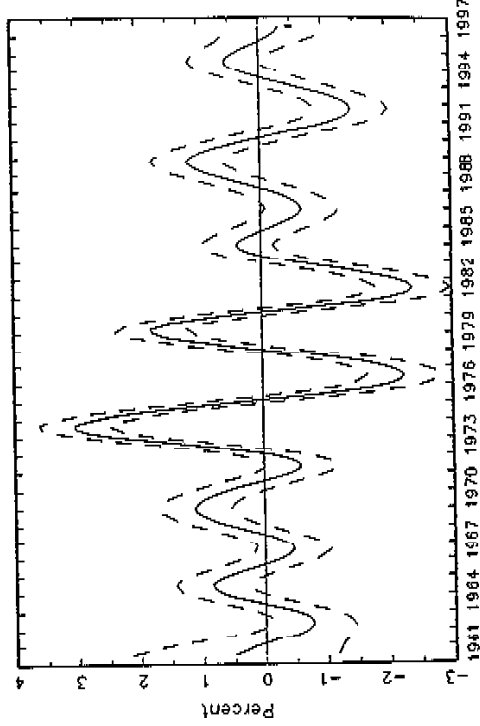
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UK: OUTPUT GAP
One SE Confidence band; One-sided estimates



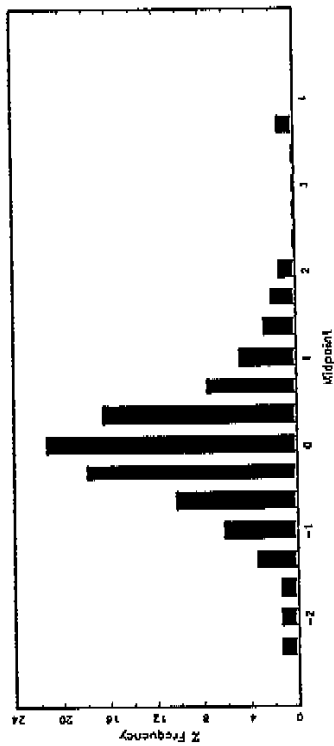
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UK: OUTPUT GAP
One SE Confidence band; Two-sided estimates



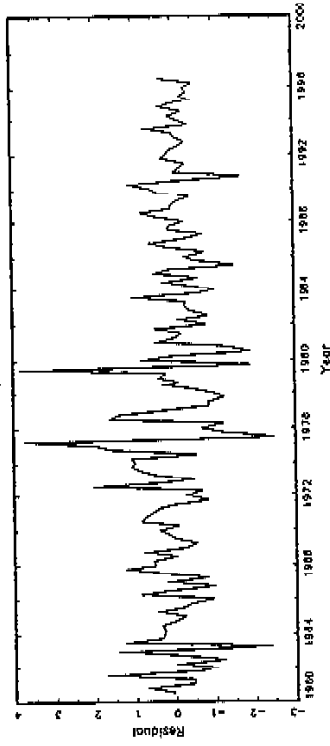
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Histogram of Inflation prediction errors



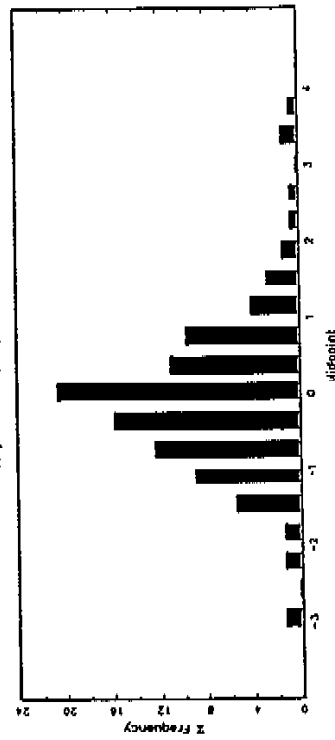
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Inflation prediction errors



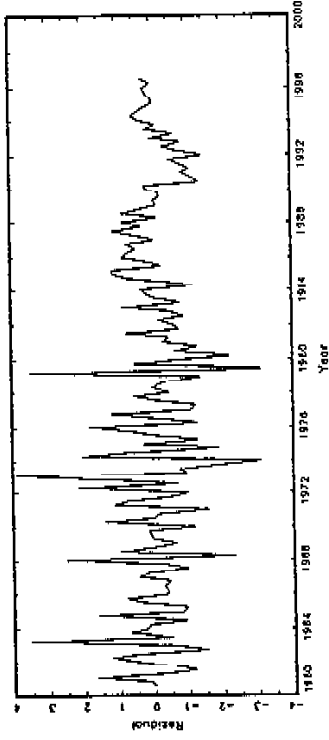
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Histogram of output prediction errors



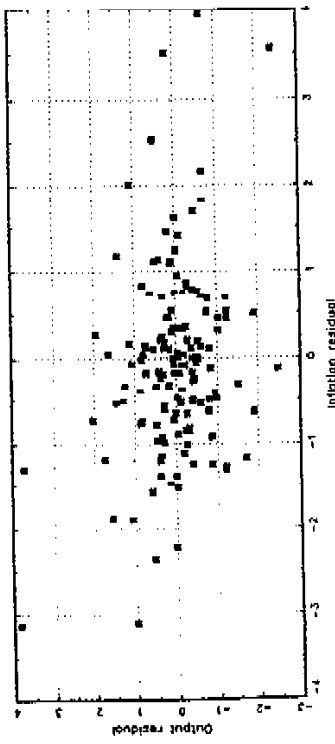
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Output prediction errors



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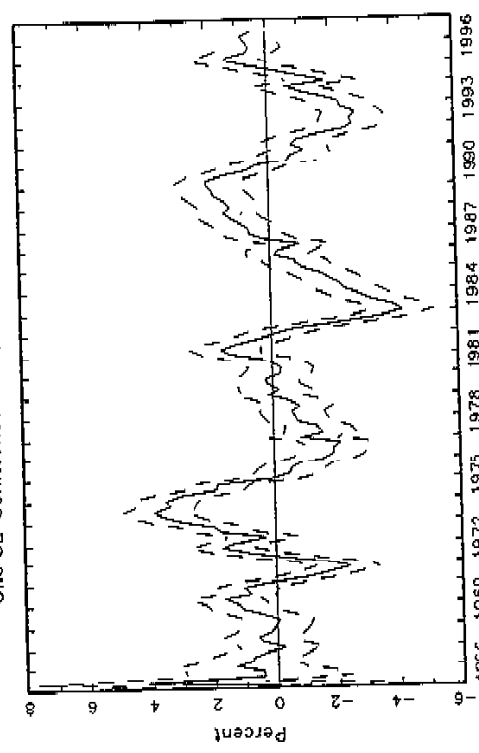
Scatterplot of prediction errors



GRAPH 8

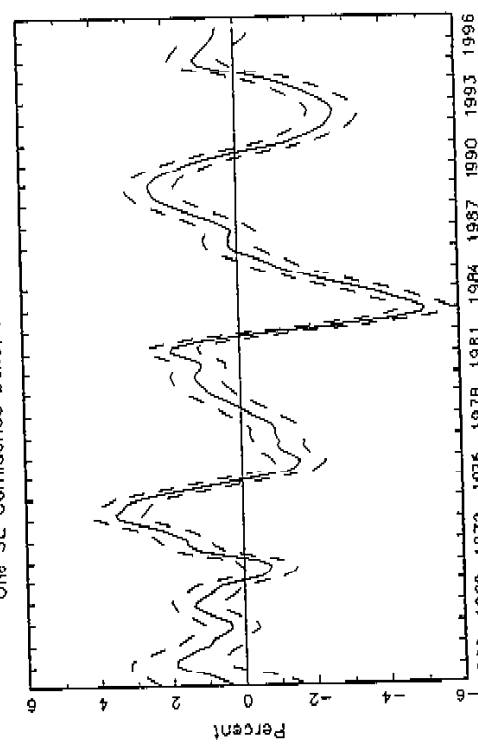
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CANADA: OUTPUT GAP
One SE Confidence band; One-sided estimates



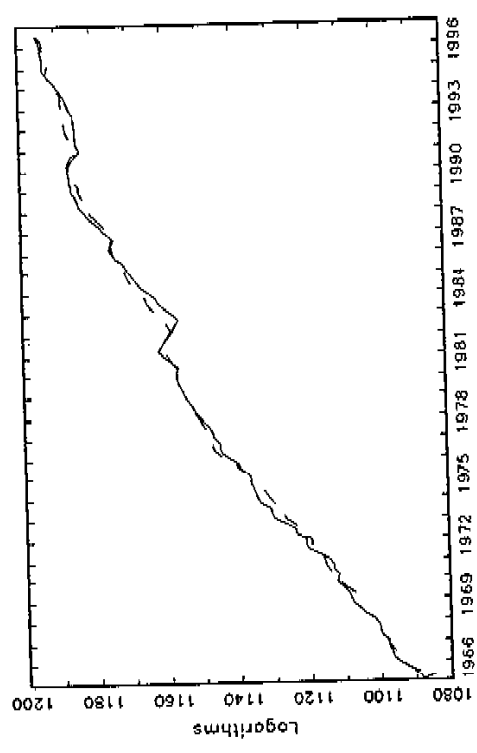
CAUTS 106 Jan 14 16:52:48 1997

CANADA: OUTPUT GAP
One SE Confidence band; Two-sided estimates



CAUTS 106 Jan 14 14:51:25 1997

CANADA: REAL GDP AND POTENTIAL
One-sided estimates



CAUTS 106 Jan 14 16:53:12 1997

CANADA: REAL GDP AND POTENTIAL
Two-sided estimates

